

الف) $\sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \geq 0 \Rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \Rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0$

$\Rightarrow \frac{x^2 - 2x + 1 - x^2}{x(x-1)} \geq 0 \Rightarrow \frac{-2x + 1}{x(x-1)} \geq 0 \rightarrow \frac{0}{+} \frac{+}{-} \frac{+}{+} \frac{-}{-} \Rightarrow Df = (-\infty, 0) \cup (\frac{1}{2}, +\infty)$

ب) $\{R - \{-2, -1\}\} \Rightarrow \{x \in R \mid x \neq -2, -1\}$
 $\{x \in R \mid \frac{3}{x-1} + \frac{1}{x+2} \neq 0\} \Rightarrow \frac{3x+2+x-1}{(x-1)(x+2)} \neq 0 \Rightarrow 4x+1 \neq 0 \Rightarrow x \neq -\frac{1}{4}$
 $\Rightarrow Df = R - \{-2, -1, -\frac{1}{4}\}$

الف) $(\frac{1}{x})^x - 9 \geq 0 \Rightarrow \frac{1}{x} \geq 9 \Rightarrow x \leq \frac{1}{9}$
 $(\frac{1}{x})^x - 9 \geq 0 \Rightarrow \frac{1}{x} \geq 9 \Rightarrow x \leq \frac{1}{9}$
 $\Rightarrow Df = (-\infty, -\frac{1}{9}] \cup [\frac{1}{9}, +\infty)$

ب) $\sqrt{x-1} + \sqrt{y+1} = 3 \Rightarrow \sqrt{x-1} + \sqrt{y+1} = 9 \Rightarrow y = 1 - \sqrt{x-1}$

$y \geq -1 \Rightarrow 1 - \sqrt{x-1} \geq -1 \Rightarrow \sqrt{x-1} \leq 2 \Rightarrow x-1 \leq 4 \Rightarrow x \leq 5$
 $x \geq 1 \Rightarrow Df = [1, 5]$

$f(x) = \frac{\log(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$
 $\begin{cases} x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0 \Rightarrow x < -1 \text{ or } x > 2 \\ x^2 - 1 > 0 \Rightarrow (x-1)(x+1) > 0 \Rightarrow x < -1 \text{ or } x > 1 \\ \sqrt{x^2 - 1} + 1 \neq 0 \Rightarrow \sqrt{x^2 - 1} \neq -1 \end{cases}$

$\Rightarrow Df = (-\infty, -1) \cup (2, +\infty)$

$\sqrt{3+an-x^2} \geq 0 \Rightarrow -(x^2 - ax - 3) \geq 0 \Rightarrow -x^2 + ax + 3 \geq 0$

$\Rightarrow -(x+2)(x-\frac{3}{2}) \geq 0 \Rightarrow -x^2 + \frac{1}{2}x - 3 \geq 0 \Rightarrow -x^2 + \frac{1}{2}x + 3 \geq 0$

$\Rightarrow a = -\frac{1}{2} \text{ و } b = 3 \Rightarrow a+b = \frac{5}{2} = 2.5$

$g(x) = \sqrt{f(x) - x} \Rightarrow f(x) - x \geq 0 \Rightarrow f(x) \geq x$

$\Rightarrow \begin{cases} 3x - 2 \geq x \Rightarrow 2x \geq 2 \Rightarrow x \geq 1 \\ 2x + 3 \geq x \Rightarrow x \geq -3 \end{cases} \Rightarrow Dg = [1, +\infty)$

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$$f(x) = f(-x) + a \Rightarrow f(a+1)(\frac{0+1}{1}) = 3a - f + a$$

$$\Rightarrow 1(a+1) = (a-f) \Rightarrow 1 \cdot a = -1 \Rightarrow a = \frac{-1}{1} = -\frac{9}{0} = -11$$

$$(1-\sqrt{3})(1+\sqrt{3}) = 1 \rightarrow \text{ریشه های معکوس} \Rightarrow 1-\sqrt{3} = a, 1+\sqrt{3} = \frac{1}{a}$$

$$\Rightarrow f(a) + f(\frac{1}{a}) = \sqrt{a} + \frac{1}{\sqrt{a}} + 1 + \frac{1}{\sqrt{a}} + \sqrt{a} + 1 = 2(\sqrt{a} + \frac{1}{\sqrt{a}}) + 2$$

$$\Rightarrow (\sqrt{a} + \frac{1}{\sqrt{a}})^2 = a + \frac{1}{a} + 2 \Rightarrow \sqrt{a} + \frac{1}{\sqrt{a}} = \sqrt{4} \Rightarrow \sqrt{4} = 2$$

$$\begin{aligned} f(x) - 3f(-x) &= 5x^2 - x \\ f(x) - 9f(-x) &= 11x^2 - 2x \\ f(-x) - 3f(x) &= 5x^2 + x \\ f(x) - 9f(-x) &= 11x^2 + 2x \end{aligned}$$

$$\Rightarrow -8f(x) = 2 \cdot x^2 + x \Rightarrow f(x) = -\frac{2x^2 + x}{8}$$

$$(x+2)f(x) - 3xf(x+2) = 5x^2 - mx + 3m - 1$$

$$\Rightarrow 4f(0) = 1 + 2m + 3m - 1 \Rightarrow 4f(0) = 10 + 8m$$

$$\Rightarrow 4f(0) = 3m - 1 \Rightarrow 4f(0) = 9m - 3$$

$$\Rightarrow 9m - 3 = 10 + 8m \Rightarrow m = 13 \Rightarrow f(0) = \frac{3m-1}{4} = \frac{39-1}{4} = \frac{38}{4} = \frac{19}{2}$$

$$\Rightarrow f(-1) + f(-1) = \frac{3(-1)^2 - 1(-1) + 3}{-1} = \frac{3+1+3}{-1} = -1$$

$$\Rightarrow f(-1) = -\frac{1}{2}$$