

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}}$ $\rightarrow \begin{cases} \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{-x^2+1}{x^2-x} \geq 0 \rightarrow \frac{1}{x} - \frac{1}{x-1} \geq 0 \\ x \neq 0, x \neq 1 \end{cases}$ $\rightarrow D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$ ✓

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{2}{x}}{\frac{2}{x-1} + \frac{1}{x+2}}$ $\rightarrow \begin{cases} x = -1, x \neq 0 \\ x \neq 1, x \neq -2 \\ \frac{2}{x-1} + \frac{1}{x+2} \neq 0 \rightarrow \frac{x+2}{(x-1)(x+2)} \neq 0 \rightarrow x \neq -\frac{2}{x} \end{cases}$ $\rightarrow D_f = \mathbb{R} - \{-2, -\frac{2}{x}, -1, 0, 1\}$ ✓

الف) $f(x) = \sqrt{((\frac{1}{x})^x - 9)(32 - 2^x)}$ $\rightarrow ((\frac{1}{x})^x - 9)(32 - 2^x) \geq 0 \rightarrow \frac{-1}{x} - \frac{2}{x} \geq 0$ $\rightarrow D_f = \mathbb{R} - (-2, 2, 8)$ ✓

ب) $\sqrt{y+1} = -\sqrt{x-1} + 3$ $\rightarrow \begin{cases} x-1 \geq 0 \rightarrow x \geq 1 \\ -\sqrt{x-1} + 3 \geq 0 \rightarrow \sqrt{x-1} \leq 3 \rightarrow x-1 \leq 9 \rightarrow x \leq 10 \end{cases}$ $\xrightarrow{\Delta} D_f = [1, 10]$ ✓

$f(x) = \frac{\log_e(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$ $\rightarrow \begin{cases} x^2 - x - 2 \geq 0 \rightarrow \frac{-1}{x} - \frac{2}{x} \geq 0 \\ x^2 - 1 \geq 0 \rightarrow x^2 \geq 1 \rightarrow x \leq -1, x \geq 1 \end{cases}$ $\rightarrow D_f = (-\infty, -1) \cup (2, +\infty) = \mathbb{R} - [-1, 2]$ ✓

$f(x) = \sqrt{-x^2 + ax + 2}$ $\rightarrow D_f = [-2, b]$ $\rightarrow x = -2 \rightarrow -4 - 2a + 2 = 0 \rightarrow a = -1, 8 \rightarrow S = \frac{-b}{a} = -1, 8 \rightarrow -2 + b = -1, 8 \rightarrow b = 1, 8$ $\rightarrow a + b = 1$

$P = \frac{C}{a} = -3 \rightarrow -2b = -3 \rightarrow b = 1, 8 \rightarrow S = 1, 8 - 2 = -1, 8 = \frac{-b}{a} \rightarrow a = -1, 8 \rightarrow 1$ ✓

$f(x) = \begin{cases} x-2; x \geq 1 \\ x+3; x < 1 \end{cases} \rightarrow g(x) = \sqrt{f(x)-x} \rightarrow f(x) \geq x$ $\rightarrow \begin{cases} x-2 \geq x; x \geq 1 \xrightarrow{\Delta} x \geq 1 \\ x+3 \geq x; x < 1 \xrightarrow{\Delta} x \leq 1 \end{cases} \rightarrow D_f = [-2, +\infty)$ ✓

$$f_{(n)} = \begin{cases} (a+1)(n+r) & ; n > 1 \\ r_a + rn & ; n \leq 1 \end{cases} \quad \text{et } f_{(0)} = f_{(-1)} + a$$

$$K(a+1) = r_a - r \rightarrow 1 \cdot a + 1 \cdot \varepsilon = \varepsilon a - \varepsilon \rightarrow 1 \cdot a = -1 \wedge \rightarrow a = -1 \wedge$$

$$f_{(n)} = \sqrt{n} + \frac{1}{\sqrt{n}} + r \rightarrow f_{(r-\sqrt{r})} + f_{(r+\sqrt{r})} = ?$$

$$= \sqrt{r-\sqrt{r}} + \frac{1}{\sqrt{r-\sqrt{r}}} + r + \sqrt{r+\sqrt{r}} + \frac{1}{\sqrt{r+\sqrt{r}}} + r = \frac{\sqrt{r-\sqrt{r}} + \sqrt{r+\sqrt{r}} + \sqrt{r+\sqrt{r}} + \sqrt{r-\sqrt{r}} + \varepsilon}{\sqrt{(r-\sqrt{r})(r+\sqrt{r})}}$$

$$= r\sqrt{r-\sqrt{r}} + r\sqrt{r+\sqrt{r}} + \varepsilon = r(\underbrace{\sqrt{r-\sqrt{r}} + \sqrt{r+\sqrt{r}}}_{\sqrt{4}} + r) = r + r\sqrt{4}$$

$$r f_{(n)} - r f_{(-n)} = \varepsilon n^r - n \xrightarrow{\times r} r f_{(n)} - r f_{(-n)} = \varepsilon n^r - rn$$

$$r f_{(-n)} - r f_{(n)} = \varepsilon n^r + n \xrightarrow{\times r} r f_{(-n)} - r f_{(n)} = \varepsilon n^r + rn$$

$$\rightarrow -\varepsilon f_{(n)} = r \cdot n^r + n \rightarrow f_{(n)} = -\frac{r n^r + n}{\varepsilon}$$

$$(n+r)(f_{(n)}) - r_n f_{(n+r)} = \varepsilon n^r - mn + r_m - 1$$

$$\xrightarrow{n=0} r f_{(0)} = r_m - 1 \rightarrow r f_{(0)} = r_m - r \quad \left. \vphantom{\begin{matrix} r f_{(0)} = r_m - 1 \\ r f_{(0)} = r_m - r \end{matrix}} \right\} \rightarrow r_m - r = r_m + 1 \cdot \delta$$

$$\xrightarrow{n=-r} r f_{(0)} = 1 \cdot \delta + r_m$$

$$r_m = 1 \wedge$$

$$\boxed{m = \varepsilon / \delta} \rightarrow f_{(0)} = 1, r \delta$$

$$f_{(n)} + f_{\left(\frac{1}{n}\right)} = \frac{r n^r - 1 r n + r}{n} \rightarrow r f_{(-1)} = \frac{r + 1 r + r}{-1} = -1 \wedge$$

$$\rightarrow f_{(-1)} = -1$$