کیان جعفری

① الف) $f(x)=\sqrt{\frac{(x-1)^2-x^2}{x^2-x}}=\sqrt{\frac{-2x+1}{x^2-x}}$

$\Rightarrow D_f=(-\infty,0)\cup[\frac{1}{2},1)$

ب) $f(x)=\frac{\frac{1}{x+1}-\frac{2}{x}}{\frac{2}{x-1}+\frac{1}{x+2}} \Rightarrow x\neq\{-1,0,1,-2\} \longrightarrow D_f=\mathbb{R}-\{\pm1,0,2,\frac{-5}{2}\}$

$\frac{2x+4+x-1}{(x-1)(x+2)}\neq 0 \Rightarrow 3x+5\neq 0 \Rightarrow x\neq\frac{-5}{3}$

② الف) $f(x)=\sqrt{\left(\left(\frac{1}{2}\right)^x-9\right)(2^{x+2}-2^x)}$

$\Rightarrow D_f=(-\infty,-2]\cup[2,3]$

ب)

③ $x^2-x-2>0 \Rightarrow$ $\Rightarrow(-\infty,-1)\cup(2,+\infty)$

$x^2-1\geq 0 \Rightarrow$ $\Rightarrow(-\infty,-1]\cup[1,+\infty)$

$\sqrt{x^2-1}+1\neq 0 \Rightarrow x^2-1\neq 1 \Rightarrow x^2\neq 2 \Rightarrow x\neq\pm\sqrt{2}$

$DF=(-\infty,-1)\cup(2,+\infty)-\{\pm\sqrt{2}\}$

④ $2+ab+b^2=0 \Rightarrow ab+b^2=+2$, $2+-2a-2=0$

$-2a=1\Rightarrow a=\frac{-1}{2}$

$\Rightarrow \frac{-b}{2}-b^2=+2 \Rightarrow -b-2b^2=4 \Rightarrow 2b^2+b+4=0$

(۸)

$$f(x)=\begin{cases}2x-1 & x\geq 1 \xrightarrow{g(x)} \sqrt{2x-1} \Rightarrow 2x\geq 2 \Rightarrow \boxed{x\geq 1}\\ 2x+2 & x<1 \xrightarrow{g(x)} \sqrt{x+2} \Rightarrow x+2\geq 0 = x\geq -2\end{cases}$$

$$\Rightarrow -2\leq x<1 \Rightarrow D_f=[-2,+\infty)$$

(۹) $2(a+1)(5)=2a-4+a \Rightarrow 12a+12=2a-4$

$$\Rightarrow 10a=-16 \Rightarrow a=\frac{-16}{10}=-1/6$$

(۷) $\sqrt{2-\sqrt{2}}=\frac{\sqrt{2}-1}{\sqrt{2}}$ و $\sqrt{2+\sqrt{2}}=\frac{\sqrt{2}+1}{\sqrt{2}}$

$$\frac{\sqrt{2}-1}{\sqrt{2}}+\frac{\sqrt{2}(\sqrt{2}+1)}{\sqrt{2}}+2+\frac{(\sqrt{2}+1)\sqrt{2}}{\sqrt{2}+2}+\frac{\sqrt{2}(\sqrt{2}-1)}{\sqrt{2}+1}+2$$

$$\Rightarrow \frac{\sqrt{2}}{2}+2=\boxed{\frac{2\sqrt{2}+4}{2}}$$

(۸)
$$\begin{cases}2f(x)-2f(-x)=2x^2-x\\ 2f(-x)-2f(x)=4x^2+x\end{cases} \Rightarrow \begin{cases}4f(x)-2f(-x)=8x^2-2x\\ 2f(-x)-9f(x)=12x^2+3x\end{cases}$$

$$-5f(x)=20x^2+x$$

$$f(x)=-4x^2-\frac{x}{5}$$

(۹) $2f(\frac{0}{m})=2m-1 \xrightarrow{\times 2} 4f(0)=8m-4$

$$4f(0)=\frac{17+2m+2m-1}{5m+10} \Rightarrow 8m-4=5m+10$$

$$2m=18 \Rightarrow m=\boxed{9}$$

$$12/5-1=11/5=f(0) \Rightarrow f(0)=2/25$$

(۱۰)
$$\frac{ax+b+\frac{1}{ax+b}}{ax+b}=\frac{3x^2-12x+2}{x}$$

$$\frac{(ax+b)^2+1}{ax+b}=a^2x^2+b^2+2abx+1$$

$$\boxed{a=1,\ b=0}$$

۱۵) $\underbrace{ax+b+\frac{a}{x}+b}_{\frac{ax^2+bx+a+bx}{x}} = \frac{3x^2-12x+3}{x} \Rightarrow \begin{matrix} a=3 \\ b=-9 \end{matrix}$

$f(x)=3x-7 \Rightarrow -3-7=-9$