

۱۵، ۷۵

Year. Month. Date. ()

امتحان ریاضیات / آزمون ریاضیات / آزمون ریاضیات

(الف) $f(x) = \frac{x-1}{x} - \frac{x}{x-1} \Rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0$ (۲) (۱)

$\Rightarrow \frac{(x-1)(x-1) - x(x)}{x(x-1)} = \frac{x^2 - 2x + 1 - x^2}{x(x-1)} = \frac{1-2x}{x(x-1)} \geq 0$
 تعیین علامت: $\frac{1}{x} - 2$
 $\frac{1}{x} - 2 \geq 0 \Rightarrow \frac{1-2x}{x} \geq 0$

$\Rightarrow D_f = (-\infty, 0) \cup \left[\frac{1}{2}, 1\right)$ ✓

(ب) $f(x) = \frac{x}{x-1} + \frac{1}{x+2} \Rightarrow \frac{x}{x-1} + \frac{1}{x+2} \geq 0$
 $\frac{x}{x-1} + \frac{1}{x+2} \geq 0 \Rightarrow \frac{x(x+2) + (x-1)}{(x-1)(x+2)} \geq 0$
 $\frac{x^2 + 2x + x - 1}{(x-1)(x+2)} \geq 0 \Rightarrow \frac{x^2 + 3x - 1}{(x-1)(x+2)} \geq 0$

$\frac{x}{x-1} + \frac{1}{x+2} \geq 0 \Rightarrow \frac{x^2 + 3x - 1}{(x-1)(x+2)} \geq 0$
 تعیین علامت: $x^2 + 3x - 1 = 0 \Rightarrow x = \frac{-3 \pm \sqrt{9+4}}{2} = \frac{-3 \pm \sqrt{13}}{2}$

$\Rightarrow D_f = \mathbb{R} - \left\{-2, -1, 0, 1, \frac{\sqrt{13}-3}{2}\right\}$ ✓

(۲) $f(x) = \left(\left(\frac{1}{x}\right)^n - 9\right)(x^2 - x^n) \geq 0 \Rightarrow (x^{-n} - 9)(x^2 - x^n) \geq 0$
 جدول تغییر علامت:
 ۱. $x^{-n} - 9 \geq 0 \Rightarrow x^{-n} \geq 9 \Rightarrow x \leq -\frac{1}{3}$
 ۲. $x^2 - x^n \geq 0 \Rightarrow x^2(1 - x^{n-2}) \geq 0 \Rightarrow 1 - x^{n-2} \geq 0 \Rightarrow x^{n-2} \leq 1$
 ۳. $x^{-n} - 9 \leq 0 \Rightarrow x^{-n} \leq 9 \Rightarrow x \geq \frac{1}{3}$
 ۴. $x^2 - x^n \leq 0 \Rightarrow x^2(1 - x^{n-2}) \leq 0 \Rightarrow 1 - x^{n-2} \leq 0 \Rightarrow x^{n-2} \geq 1$

$\Rightarrow D_f = \left\{-2, \frac{5}{2}\right\} \cup \mathbb{R} - \left(-2, \frac{5}{2}\right)$

(ب) $\sqrt{x-1} + \sqrt{y+1} = 3$

$x-1 \geq 0 \Rightarrow x \geq 1 \rightarrow x=1 \Rightarrow y=8$
 $y+1 \geq 0 \Rightarrow y \geq -1 \rightarrow y=-1 \Rightarrow x=12$
 $\Rightarrow D_f = [1, 12]$ ✓

$$f(x) = \begin{cases} (a+1)(x+2) & ; x > 1 \\ 3a + 2x & ; x \leq 1 \end{cases}$$

$$2f(8) = f(-2) + a$$

(2)

(9)

$$f(8) \rightarrow f(8) = (a+1)(8) \Rightarrow 2f(8) = 16a + 16$$

$$f(-2) \rightarrow f(-2) = 3a - 2$$

$$\Rightarrow 16a + 16 = 3a - 2 \Rightarrow a = -18$$

(1)

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + x, \quad f(x-\sqrt{x}) + f(x+\sqrt{x})$$

(2)

$$\Rightarrow f(x-\sqrt{x}) = \sqrt{x-\sqrt{x}} + \frac{1}{\sqrt{x-\sqrt{x}}} + x$$

$$\Rightarrow f(x+\sqrt{x}) = \sqrt{x+\sqrt{x}} + \frac{1}{\sqrt{x+\sqrt{x}}} + x$$

$$\Rightarrow 2\sqrt{x-\sqrt{x}} + 2\sqrt{x+\sqrt{x}} + 2x$$

$$\Rightarrow 2(\sqrt{x-\sqrt{x}} + \sqrt{x+\sqrt{x}}) + 2x = 2(\sqrt{4}) + 2x = 4 + 2x$$

$$\Rightarrow f = 2\sqrt{4} + 2x$$

$$f(x) - f(-x) = \epsilon x^2 - x$$

$$f(x) - f(-x) = \epsilon x^2 + x$$

(15)

$$f(x) - f(-x) = \epsilon x^2 - x \Rightarrow \epsilon x^2 - x = 0 \Rightarrow x = \frac{1}{\epsilon}$$

این به باطل می رسد

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$$f(x) = 0 \Rightarrow f(0) = 0$$

$$f(x) = 0$$

$$(n+2)f(n) - 3nf(n+2) = \varepsilon n^2 - mn + 3m - 1 \quad (4)$$

$$f(0) \Rightarrow n=0 \Rightarrow 2f(0) - 3(0)f(2) = \varepsilon(0)^2 - m(0) + 3m - 1$$

$$\Rightarrow 2f(0) = 3m - 1$$

$$\cancel{m(0) = 0 \Rightarrow 2f(0) = 3m - 1}$$

$$n=1 \rightarrow n=-2 \rightarrow 0(f(-2)) + 7f(0) = 14 + 3m - 1 = 13 + 3m$$

$$\Rightarrow 7f(0) = 13 + 3m, 2f(0) = 3m - 1 \Rightarrow \frac{13}{5} + \frac{3m}{5} = \frac{3m - 1}{2} \Rightarrow m = \frac{11}{\varepsilon}$$

$$\text{جواب: } 2f(0) = \frac{3 \times 11}{\varepsilon} - 1 = \frac{33}{\varepsilon} - 1 \Rightarrow \boxed{f(0) = \frac{17}{\varepsilon}} \quad \checkmark$$

$$f \rightarrow \text{تابع } f(n) + f\left(\frac{1}{n}\right) = \frac{\varepsilon n^2 - 12n + 9}{n}, f(-1) \quad (10)$$

$$\cancel{f(-1) + f(1) = \frac{\varepsilon(-1)^2 - 12(-1) + 9}{-1} = \varepsilon + 3} \quad (11)$$

$$f(n) + f\left(\frac{1}{n}\right) = an + b + a\left(\frac{1}{n}\right) + b = an + \frac{a}{n} + 2b \quad \checkmark$$

$$\frac{\varepsilon n^2 - 12n + 9}{n} = \frac{n^2 - 12n + 9}{n} \Rightarrow n - 12 + \frac{9}{n} \rightarrow n = -1 \rightarrow (-12 - 9) = -21$$

$$n = -1 \rightarrow 2f(-1) = -21 \rightarrow f(-1) = -9$$