Year. Month. Date. ( )

امیرحسین یزدخواستی — یازدهم ریاضی — تکلیف شماره ۱

(۱)

الف) $f(x)=\sqrt{\dfrac{x-1}{x}-\dfrac{x}{x-1}} \Rightarrow \dfrac{x-1}{x}-\dfrac{x}{x-1}\geq 0$

$\Rightarrow \dfrac{(x-1)(x-1)-x(x)}{x(x-1)}=\dfrac{x^2-2x+1-x^2}{x(x-1)}=\dfrac{1-2x}{x(x-1)}\geq 0 \xrightarrow{\text{تعیین علامت}}$

x		0		$\frac{1}{2}$		1	
	$+$		$-$		$+$		$-$

$\Rightarrow D_f=(-\infty,0)\cup[\frac{1}{2},1)$

ب) $f(x)=\dfrac{\dfrac{1}{x+2}-\dfrac{2}{x}}{\dfrac{3}{x-1}+\dfrac{1}{x+2}} \Rightarrow \dfrac{\dfrac{-x-2}{x(x+2)}}{\dfrac{3x+6+x-1}{(x-1)(x+2)}}=\dfrac{\dfrac{-x-2}{x(x+2)}}{\dfrac{4x+5}{(x-1)(x+2)}}$

$-1,\ 0,\ 1,\ -2$

$\dfrac{3}{x-1}+\dfrac{1}{x+2}\neq 0 \Rightarrow \dfrac{3x+6+x-1}{(x-1)(x+2)}=\dfrac{4x-5}{(x-1)(x+2)} \xrightarrow{\text{ریشه‌یابی}} 4x=5 \Rightarrow x=\dfrac{5}{4}$

$\Rightarrow D_f=\mathbb{R}-\{-2,-1,0,1,\frac{5}{4}\}$

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(۲)

الف) $f(x)=\sqrt{\left((\frac{1}{3})^x-9\right)(32-4^x)} \Rightarrow (3^{-x}-3^2)(2^5-2^{2x})\geq 0$

هر دو پرانتز بزرگتر مساوی صفر:
$3^{-x}-3^2\geq 0 \rightarrow 3^{-x}\geq 3^2 \Rightarrow \boxed{x\geq -2}$
$2^5-2^{2x}\geq 0 \rightarrow 2^5\geq 2^{2x} \Rightarrow \boxed{x\geq \frac{5}{2}}$

یا

هر دو پرانتز کوچکتر مساوی صفر:
$3^{-x}-3^2\leq 0 \Rightarrow 3^{-x}\leq 3^2 \Rightarrow \boxed{x\geq -2}$
$2^5-2^{2x}\leq 0 \Rightarrow 2^5\leq 2^{2x} \Rightarrow \boxed{x\leq \frac{5}{2}}$

$\Rightarrow D_f=\{-2,\frac{5}{2}\}$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3$

$x-1\geq 0 \Rightarrow x\geq 1 \longrightarrow x=1 \Rightarrow y=8$
$y+1\geq 0 \Rightarrow y\geq -1 \longrightarrow y=-1 \Rightarrow x=82$
$\Rightarrow D_f=[1,82]$

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Year. Month. Date. ( )

امیدحسین یزدخواستی پایه دهم ۸ تجربی — تمرین شماره ٤

۳)

$$f(x)=\frac{\log_{4}^{(x^2-x-2)}}{\sqrt{x^2-1}+1} \longrightarrow \log_b^a = c \Rightarrow \begin{matrix} a>0 \\ b>0 \\ b\neq 1 \end{matrix}$$

$\mathrm{I} \Rightarrow x^2-x-2>0 \Rightarrow (x-2)(x+1) \longrightarrow \begin{matrix} -1 & & 2 \\ + \ 0 & - & 0 \ + \end{matrix} \Rightarrow D_f=(-\infty,-1)\cup(2,+\infty)$

$\mathrm{II} \Rightarrow \sqrt{x^2-1} \longrightarrow x^2-1\geq 0 \Rightarrow (x-1)(x+1)\geq 0 \longrightarrow \begin{matrix} -1 & & +1 \\ + \ 0 & - & 0 \ + \end{matrix} \Rightarrow D_f=(-\infty,-1]\cup[+1,+\infty)$

$\mathrm{III} \Rightarrow \sqrt{x^2-1}+1\neq 0 \Rightarrow \sqrt{x^2-1}\neq -1 \longrightarrow D_f=\mathbb{R}$ ← زیرا که همواره مقدار حاصل رادیکال وار مقدار مطلق بزرگتر و مثبت است

$\xrightarrow{\cap} (-\infty,-1)\cup(2,+\infty)=D_f$

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٤)

$f(x)=\sqrt{3+ax-x^2}\ , \ D_{f(x)}=[-2,b]$

$\xrightarrow{x=-2 \text{ جایگذاری }} 3-2a-4=-1-2a=0 \Rightarrow 2a=-1 \Rightarrow \boxed{a=\frac{-1}{2}}$

$\Rightarrow f(x)=\sqrt{3-\frac{1}{2}x-x^2} \longrightarrow -x^2-\frac{1}{2}x+3\geq 0 \xrightarrow{\text{تعیین علامت}} \begin{matrix} -2 & & \frac{3}{2} \\ - \ 0 & + & 0 \ - \end{matrix}$

$\Rightarrow D_f=[-2,\frac{3}{2}] \Rightarrow \boxed{b=\frac{3}{2}} \Rightarrow \boxed{a+b=\frac{-1}{2}+\frac{3}{2}=\frac{2}{2}=1}$

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۵)

$$f(x)=\begin{cases} 3x-2 \ ; & x\geq 1 \\ 2x+3 \ ; & x<1 \end{cases} \ , \ g(x)=\sqrt{f(x)-x}$$

$D_{g(x)}$:

$x\geq 1 \Longrightarrow g(x)=\sqrt{3x-2-x}=\sqrt{2x-2} \Rightarrow 2x-2\geq 0 \Rightarrow \boxed{x\geq 1}$

$x<1 \Rightarrow g(x)=\sqrt{2x+3-x}=\sqrt{x+3} \Rightarrow x+3\geq 0 \Rightarrow \boxed{x\geq -3}$

$\xrightarrow{\cap} \boxed{D_f=[1,+\infty)}$

Year. Month. Date. ( )

امیرحسین یزدخواستی — یازدهم سه A تجربی — تکلیف شماره ۴

۶)

$$f(x)=\begin{cases}(a+1)(x+2) & ; x>1\\ 3a+2x & ; x\le 1\end{cases} \quad , \quad 2f(5)=f(-2)+a$$

$$f(5)\longrightarrow f(5)=(a+1)(7) \Rightarrow 2f(5)=14a+14$$

$$f(-2)\longrightarrow f(-2)=3a-4$$

$$\Rightarrow 14a+14=3a-4+a$$

$$\Rightarrow 10a+18=0 \Rightarrow \boxed{a=-1.8}$$

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۷)

$$f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2 \quad , \quad f(2-\sqrt{3})+f(2+\sqrt{3})$$

$$\times\frac{\sqrt{2+\sqrt{3}}}{\sqrt{2+\sqrt{3}}}=\frac{\sqrt{2+\sqrt{3}}}{1}$$

$$\Rightarrow f(2-\sqrt{3})=\sqrt{2-\sqrt{3}}+\frac{1}{\sqrt{2-\sqrt{3}}}+2$$

$$\frac{\sqrt{2-\sqrt{3}}}{1}$$

$$\Rightarrow f(2+\sqrt{3})=\sqrt{2+\sqrt{3}}+\frac{1}{\sqrt{2+\sqrt{3}}}+2$$

$$\Rightarrow 2\sqrt{2-\sqrt{3}}+2\sqrt{2+\sqrt{3}}+4$$

$$\Rightarrow 2(\underbrace{\sqrt{2-\sqrt{3}}}_{a}+\underbrace{\sqrt{2+\sqrt{3}}}_{b})+4 \longrightarrow (a+b)^2=2-\cancel{\sqrt{3}}+2+\cancel{\sqrt{3}}+2=6 \Rightarrow \boxed{a+b=\sqrt{6}}$$

$$\Rightarrow f=\boxed{2\sqrt{6}+4}$$

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۸)

$$+\begin{cases}2f(x)-3f(-x)=4x^2-x\\ 2f(-x)-3f(x)=4x^2-x\end{cases}$$

$$-f(x)-f(-x)=8x^2-2x \Rightarrow 8x^2-2x=0 \Rightarrow \boxed{x=\frac{1}{4},0}$$

$$4\times\frac{1}{16}-\frac{1}{4}=0$$

$$\longrightarrow 2f(\tfrac{1}{4})-3f(-\tfrac{1}{4})=0$$

$$2f(-\tfrac{1}{4})-3f(\tfrac{1}{4})=0 \quad \Rightarrow f(\tfrac{1}{4})=0$$

جایگذاری

$$\longrightarrow x=\frac{1}{4} \longrightarrow f(x)=0$$

$$\longrightarrow x=0 \longrightarrow f(x)=0 \longrightarrow 2f(0)-3f(0)=0 \Rightarrow f(0)=0$$

$$\Rightarrow \boxed{f(x)=0}$$

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Year. Month. Date. ( )

امیرحسین فرخزادی یازدهم سیاه کتاب کاملاً شفاف!

(۹) $(x+2)f(x) - 3xf(x+2) = 4x^2 - mx + 3m - 1$

$f(0) \Rightarrow x=0 \Rightarrow 2f(0) - \cancel{3(0)f(2)} = 2f(0) = \cancel{4(0)^2} - \cancel{m(0)} + 3m - 1$

$\Rightarrow 2f(0) = 3m - 1$

$\xrightarrow{\text{با}} x=-2 \longrightarrow 0(\cancel{f(-2)}) + 6f(0) = 16 + 5m - 1 = 15 + 5m$

$\Rightarrow 6f(0) = 15 + 5m \;,\; 2f(0) = 3m - 1 \Rightarrow \overset{18}{\cancel{15}} + \overset{4m}{\cancel{5m}} = 9m - \cancel{3} \Rightarrow m = \frac{18}{4}$

جایگذاری $\Rightarrow 2f(0) = \frac{3 \times 18}{4} - 1 = \frac{50}{4} \Rightarrow \boxed{f(0) = \frac{25}{4}}$

(۱۰) $f \longrightarrow$ تابع خطی $\;,\; f(x) + f(\frac{1}{x}) = \frac{3x^2 - 12x + 3}{x} \;,\; f(-1) = ?$

$f(x) + f(\frac{1}{x}) = ax + b + a(\frac{1}{x}) + b = ax + \frac{a}{x} + 2b$

$\frac{3x^2 - 12x + 3}{x} = \frac{x^2 - 12x + 9}{x} \Rightarrow x - 12 + \frac{9}{x} \longrightarrow x = -1 \longrightarrow -1 - 12 - 9 = \boxed{-22}$