نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

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الف) $f(x)=\frac{x-1}{x}\cdot\frac{x}{x-1} \Rightarrow \frac{x-1}{x}\cdot\frac{x}{x-1}\geq 0 \quad \frac{(x-1)^2-x^2}{x(x-1)}\geq 0$

$\frac{(x-1-x)(x-1+x)}{x(x-1)}\geq 0 \quad \frac{(2x-1)(-1)}{x(x-1)}\geq 0$

ب) $f(x)=\frac{\frac{1}{x+1}-\frac{2}{x}}{\frac{4}{x-1}+\frac{1}{x+2}} = \frac{\frac{x-2x-2}{x(x+1)}}{\frac{4x+4+x-1}{(x-1)(x+2)}} = \frac{\frac{x+2}{x(x+1)}}{\frac{5x+3}{(x-1)(x+2)}} \cdot \frac{(x+2)^2(x-1)}{x(x+1)(5x+3)}$

$D_f=\mathbb{R}-\{-2,-\frac{3}{5},-1,0,1\}$

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الف) $f(x)=\sqrt{(\frac{1}{2})^x-9}\sqrt{2^{x+2}-4^x}$

x	$-\infty$		-2		3		$+\infty$
$(\frac{1}{2})^x-9$		$+$	0	$-$		$-$	
$2^{x+2}-4^x$		$-$		$+$	0	$-$	
$f(x)$		$+$	0	$-$	0	$+$	

$D_f=(-\infty,-2]\cup[3,+\infty)$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3 \quad \sqrt{y+1}=3-\sqrt[4]{x-1} \quad x\geq 1 \quad 3-\sqrt[4]{x-1}\geq 0$

$\sqrt[4]{x-1}\leq 3 \quad x-1\leq 81 \quad x\leq 82 \quad D=[1,82]$

۳

$x^2-x-2>0 \quad (x-2)(x+1)>0$ (+ | − | +) $\quad x<-1 \quad x>2$

$x^2-1\geq 0 \quad (x-1)(x+1)\geq 0$ (+ | − | +) $\quad x\leq -1 \quad x\geq 1$

$\sqrt{x^2-1}+1\neq 0 \quad \sqrt{x^2-1}\neq -1 \quad x\in\mathbb{R} \Rightarrow D=(-\infty,-1)\cup(2,+\infty)$

۴

$2-2a-4=0 \quad 2a=-1 \quad a=-\frac{1}{2} \quad \sqrt{-x^2-\frac{1}{2}x+3}=f(x)$

$\Delta=\frac{1}{4}+12=12,25 \quad \frac{\frac{1}{2}+3,5}{-2}=-2 \quad \frac{\frac{1}{2}-3,5}{-2}=1,5=b$

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$g(x)\begin{cases}\sqrt{2x-2} & x\geq 1 \rightarrow \text{جذر نمی‌تواند منفی شود.}\\ \sqrt{x+3} & x<1 \rightarrow x\geq -3\end{cases} \quad D_g=[-3,+\infty)$

$f(a) = (a+1)(a+2) = a^2 + 3a + 2 \quad f(-2) = 2a - 1$

$2a + 1 = a^2 - 1 \Rightarrow a^2 - 2a - 2 = 0 \Rightarrow a = -1, 3$ ۶

$(2+\sqrt{2})(2-\sqrt{2}) = 1 \Rightarrow 2+\sqrt{2} = \frac{1}{2-\sqrt{2}} \Rightarrow f(2+\sqrt{2}) = f(2-\sqrt{2}) = 2f(2+\sqrt{2})$

$3 + \sqrt{2} = (\sqrt{m}+\sqrt{n})^2 = m+n+2\sqrt{mn} \Rightarrow m+n=3 \quad 2\sqrt{mn} = \sqrt{2} \Rightarrow mn = \frac{1}{8}$

$(t-m)(t-n) = 0 \Rightarrow$ مجموع ریشه‌ها $\Rightarrow t^2 - (m+n)t + mn = 0 \quad t^2 - 3t + \frac{1}{8} = 0$ ۷

$t = \frac{3 \pm \sqrt{9-2}}{2} = \frac{1}{2}, \frac{7}{2} = n, m \Rightarrow 2+\sqrt{2} = (\sqrt{\frac{1}{2}} + \sqrt{\frac{7}{2}})^2 = (\frac{\sqrt{7}+\sqrt{1}}{2})^2 \Rightarrow \frac{1}{2+\sqrt{2}} = (\frac{\sqrt{7}-\sqrt{1}}{2})^2$

$\Rightarrow 2f(2+\sqrt{2}) = 2(\frac{\sqrt{7}+\sqrt{1}}{2}, \frac{\sqrt{7}-\sqrt{1}}{2}, 2) = \sqrt{7}+\sqrt{2}+\sqrt{7}-\sqrt{2}+2 = 4+2\sqrt{7}$

$f(0) = 0$

$2f(1) - 3f(-1) = 2$

$2f(1) + 2f(-1) = 8$

$f(1) = -\frac{-8-2}{4-a} = -\frac{21}{8}$

$f(-1) = \frac{10-9}{4-a} = -\frac{19}{8}$

$f(x) = ax^2 + bx + c$ $\quad f(x) = -4x^2 - 0.2x$

$f(0) = 0 \Rightarrow c = 0$

$f(1) = a+b = -\frac{21}{8}$

$f(-1) = a-b = -\frac{19}{8}$

$f(1) + f(-1) = 2a = -\frac{40}{8} = -5$

$\Rightarrow a = -4 \Rightarrow b = -0.12$ ۸

$x=0 \Rightarrow 2f(0) = 3m+1 \xrightarrow{\times 2} 4f(0) = 6m+2$

$x=-2 \Rightarrow 4f(0) = 14+2m+3m+1 = 5m+15$

$\Rightarrow 6m+2 = 5m+15$

$4m = 12$

$m = 3, 0$ ۹

$ax+b+\frac{a}{x}+b = ax+2b+\frac{a}{x} = \frac{ax^2+2bx+a}{x} = \frac{3x^2-12x+2}{x}$

$\Rightarrow a = 3 \quad b = -4 \quad f(x) = 3x - 4 \quad x = -1 \Rightarrow f(x) = -7$ ۱۰