

$$\left. \begin{aligned} x^y \left(\left(\frac{1}{y} \right)^{x+1-y} - 1 \right) &\geq 0 \Rightarrow x^y \left(\left(\frac{1}{y} \right)^{x-1} - 1 \right) \geq 0 \\ \left(\frac{1}{y} \right)^x &\geq 1 \Leftrightarrow x \leq 0 \end{aligned} \right\} \Rightarrow \begin{array}{c} x \mid \begin{array}{ccc} 0 & 1 & - \\ - & + & - \end{array} \\ \boxed{x \in [0, 1]} \end{array}$$

$$f(-x) = \sqrt{-x + |y-x|} \Rightarrow |y-x| - x \geq 0 \begin{cases} x \leq y \Rightarrow y-x-x \geq 0 \Rightarrow y \geq 2x \Rightarrow 1 \geq 2x \Rightarrow \boxed{1 \leq x} \\ x > y \Rightarrow x-y-x \geq 0 \Rightarrow -y \geq 0 \quad \times \end{cases} \Rightarrow \boxed{x \in [1, +\infty)}$$

$$\frac{x-1}{f(x)} \geq 0 \quad x \mid \begin{array}{cccc} - & + & 1 & 2 & 3 \\ 0 & + & 0 & - & 0 & + \end{array} \Rightarrow x \in \underset{(I)}{(-\infty, 1] \cup (y, 3)} \xrightarrow{\text{I} \cap \text{II}} \boxed{x \in (-\infty, 1] \cup (y, 3)}$$

$\Rightarrow x \in D_f \Rightarrow x \in [-\infty, 3] \quad (I \cup II)$

$$\begin{aligned} 9x^6 - 12x^5 - 5x^4 - 3 &= 9x^6 + 9x^5 + 1 - x^5 - 5x^4 - 3 = (3x^3 - 1)^2 - (x+2)^2 = \\ (3x^3 - 1 + x + 2)(3x^3 - 1 - x - 2) &= (3x^3 + x + 1)(3x^3 - x - 3) \Rightarrow D_{f_1} = \mathbb{R} - \{P(x) \text{ های ریشه های } \} \\ &= D_{f_2} = \mathbb{R} - \{Q(x) \text{ های ریشه های } \} \\ \Rightarrow \underbrace{3x^3 + ax + b}_{Q(x)} &= 3x^3 - x - 3 \\ \Rightarrow \boxed{a = -1, b = -3} &\Rightarrow \sqrt{-(a+b)} = \sqrt{-(-1-3)} = \sqrt{4} = \boxed{2} \end{aligned}$$

$$\begin{aligned} x \neq 0, f(x) - f\left(\frac{x}{2}\right) &\geq 0 \Rightarrow x^y + x - \left(\frac{x}{2}\right)^y - \frac{x}{2} \geq 0 \\ \Rightarrow x - \frac{x}{2} + x^y - \left(\frac{x}{2}\right)^y &= \left(x - \frac{x}{2}\right) + \left(x - \frac{x}{2}\right) \left(x^y + x\left(\frac{x}{2}\right) + \left(\frac{x}{2}\right)^y\right) = \left(x - \frac{x}{2}\right) \left(1 + x^y + x + \frac{1}{2}x^y\right) \geq 0 \\ \Rightarrow \left(x - \frac{x}{2}\right) \left(x^y + x + \frac{1}{2}x^y\right) &\geq 0 \Rightarrow \left(\frac{x-y}{x}\right) \left(\frac{x^y + x + \frac{1}{2}x^y}{x^y}\right) \geq 0 \Rightarrow \frac{(x-y)(x+y)}{x^y} \geq 0 \\ \Rightarrow x \mid \begin{array}{ccc} - & 0 & + \\ - & + & - \end{array} &\Rightarrow \boxed{x \in [-y, 0) \cup [y, +\infty)} \end{aligned}$$

$$x^3 + x = 10 \Rightarrow x = 4 \Rightarrow f(10) = f(4^3 + 4) = 4 \times 4^4 - 1 = \boxed{V}$$

$$x^3 + x = 7 \Rightarrow x = 1 \Rightarrow f(7) = f(1^3 + 1) = 1 \times 1^4 - 1 = \boxed{I}$$

$$f(10) + f(7) = 4 + 1 = \boxed{\wedge}$$

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$$g(x) = x^3 + 9x^2 + 14x + 14 - 14 = (x+3)^3 - 14$$

$$f(x) = x^3 - 6x^2 + 14x - 14 + 14 = (x-2)^3 + 14$$

$$\frac{g(\sqrt[3]{14} - 3)}{f(\sqrt[3]{14} + 2)} = \frac{(\sqrt[3]{14} - 3 + 3)^3 - 14}{(\sqrt[3]{14} + 2 - 2)^3 + 14} = \frac{14 - 14}{14 + 14} = \frac{-14}{28} = \boxed{-\frac{1}{2}}$$

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$$f(x) + g(x) = \sqrt{(x-f+y)^2} + \sqrt{(x-f-y)^2} = |\sqrt{x-f} + y| + |\sqrt{x-f} - y|$$

$$x \geq x \geq f \Rightarrow f(x) + g(x) = \sqrt{x-f} + y + y - \sqrt{x-f} = f \Rightarrow \begin{cases} c=0 \\ c \neq 0 \end{cases} \cdot \times$$

$$x \geq f \Rightarrow f(x) + g(x) = \sqrt{x-f} + y + \sqrt{x-f} - y = 2\sqrt{x-f} \Rightarrow a=0, b=y, c=-f$$

$$\Rightarrow \frac{a+b}{c} = \frac{y}{-f} = \boxed{\frac{-1}{y}}$$

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$$x^2 - 5x + 3 \neq 0 \Rightarrow (x-3)(x-1) \neq 0 \Rightarrow x \neq 1, 3$$

$$f(x) \neq 0 \Rightarrow x \neq -2$$

$$\Rightarrow D_{\frac{g}{f}} = \{0, 1, 2, 3\} \cap \{x \in \mathbb{R} : x \neq 1, 3\} - \{-2\} = \{0\}$$

$$f(0) = \frac{2}{3}$$

$$\Rightarrow \frac{g}{f} = \left\{ \left(0, \frac{2}{3} \right) \right\} = \boxed{\left\{ \left(0, \frac{2}{3} \right) \right\}}$$

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$$\text{الف) } \{(0, \omega, 4), (1, \omega, -1), (2, 2), (-\omega, \omega, 4)\} \quad \begin{aligned} f(x) &= (a, b), \dots \\ f(4x) &= \left(\frac{a}{4}, b\right), \dots \end{aligned}$$

$$\text{ب) } x \geq 0 \Rightarrow f(x^2) = \{(1, 2), (-1, 2), (\sqrt{2}, -1), (-\sqrt{2}, -1), (2, 2), (-2, 2)\} \quad \begin{aligned} f(x) &= (a, b) \\ f(x^2) &= (\sqrt{a}, b) \end{aligned} \quad \begin{aligned} x^2 &= -1 \\ x &\in \mathbb{R} \end{aligned} \cdot \times$$

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$$\text{ج) } \{(-2, 2 \times 3^2 + 1), (1, 2 \times (-1)^2 + 1), (3, 2 \times 4^2 + 1), (-1, 2 \times 0^2 + 1)\} = \{(-2, 19), (1, 3), (3, 49), (-1, 1)\}$$

$$\text{د) } D_{\frac{f}{g}} = D_f \cap D_g - \{x \mid g(x) = 0\} = \{1, 3, -1\} - \{-1\} = \{1, 3\}$$

$$\frac{f}{g} = \left\{ \left(1, \frac{3 \times 1}{1} \right), \left(3, \frac{3 \times 3}{3} \right) \right\} = \{(1, 3), (3, 3)\}$$