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$$f(x) = \left(\frac{1}{x}\right)^{x-2} \rightarrow f(x+1) = \left(\frac{1}{x+1}\right)^{x-1}$$

دقت به صورت سوال!

$$\left(\left(\frac{1}{x}\right)^{x-1} - 1\right)x^x \geq 0 \quad x^x \geq 0 \Rightarrow x \geq 0 \quad D_f = [0, +\infty)$$

$$\downarrow$$

$$x=1 \quad \text{ریشه}$$

$$\frac{0}{-1+1-}$$

$$D = [0, 1]$$

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$$\sqrt{x+1} + x + 1 \geq 0 \quad -x+1 - x+1 \geq 0$$

$$\begin{cases} x \leq 2 \rightarrow -x+2-x \geq 0 & -2x+2 \geq 0 & 2x \leq 2 & \boxed{x \leq 1} \quad \checkmark \\ x > 2 \rightarrow -x+x-2 \geq 0 \Rightarrow -2 \geq 0 & \text{ناممکن} \end{cases}$$

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$$\begin{array}{c} \rightarrow 1 \\ x-1 \\ f(x) \\ \downarrow \\ 2, 3, 4 \end{array}$$

$$\begin{array}{ccccccc} -4 & & 1 & & 2 & & 3 \\ - & + & - & + & - & + & - \end{array}$$

$$(-4, 1] \cup (2, 3) \quad \checkmark$$

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$$9x^4 - 7x^2 - 4x - 3 \Rightarrow (3x^2 - x - 3) \underbrace{(3x^2 + x + 1)}_{\mathbb{R}}$$

$$D_{f_1} = D_{f_2} \rightarrow 3x^2 - x - 3 = 3x^2 + ax + b \quad \begin{matrix} a = -1 \\ b = -3 \end{matrix} \quad \sqrt{-(a+b)} \Rightarrow \sqrt{2} = 2 \quad \checkmark$$

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$$f(x) - f\left(\frac{x}{2}\right) \geq 0$$

$$\begin{cases} f(x) \geq f\left(\frac{x}{2}\right) \quad x > 0 \rightarrow x \geq 2 \\ f(x) \geq f\left(\frac{x}{2}\right) \quad x < 0 \rightarrow x \geq -2 \end{cases}$$

$$D_f = [-2, 0) \cup [2, +\infty) \quad \checkmark$$

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$$\left. \begin{aligned} 1^3 + 1 &= 2 \Rightarrow f(1) \\ 2^3 + 2 &= 10 \Rightarrow f(2) \end{aligned} \right\} \Rightarrow \left. \begin{aligned} f(1^3+1) &= 2(1^2) - 1 = 1 \\ f(2^3+2) &= 2(2^2) - 1 = 7 \end{aligned} \right\} \Rightarrow \wedge \checkmark$$

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$$\begin{aligned} x^3 + 9x^2 + 14x &= (x+3)^3 - 27 \\ x^3 - 9x^2 + 14x &= (x-3)^3 - 27 \end{aligned}$$

$$\frac{(\sqrt[3]{x} - 3 + 3)^3 - 27}{(\sqrt[3]{x} + 3 - 3)^3 - 27} = \frac{1 - 27}{8 - 27} = \frac{-26}{-19} = \frac{26}{19} \checkmark$$

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$$\begin{aligned} \sqrt{x+4}\sqrt{x-4} &= \sqrt{(\sqrt{x-4}+2)^2} = |\sqrt{x-4}+2| \xrightarrow{x \geq 4} \sqrt{x-4}+2 \\ \sqrt{x-4}\sqrt{x-4} &= \sqrt{(\sqrt{x-4}-2)^2} = |\sqrt{x-4}-2| \xrightarrow{x \geq 4} \sqrt{x-4}-2 \end{aligned}$$

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$$f(x) + g(x) = 2\sqrt{x-4} = a + b\sqrt{x+c} \rightarrow a=0, b=2, c=-4 \quad \frac{a+b}{c} = \frac{0+2}{-4} = -\frac{1}{2} \checkmark$$

$$\frac{x+2}{x^2-5x+3} \rightarrow x \neq 1, 3 \quad \frac{g}{f} = (0, 4), (2, -\frac{1}{5}) \checkmark$$

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$$\begin{aligned} f(2) &= \frac{2+2}{2-1+3} = -2 & f(3) &= \frac{3+2}{9-15+3} = \frac{5}{0} \Rightarrow f(0) = \frac{2}{3} \\ f(1) &= \frac{1+2}{1-5+3} = \frac{3}{-1} = -3 \end{aligned}$$

$$f(2x) = \left\{ \left(\frac{1}{4}, 4\right), \left(\frac{3}{4}, -1\right), (2, 2), \left(-\frac{1}{4}, 4\right) \right\} \checkmark$$

$$f(x^2) = \left\{ (1, 2), (\sqrt{2}, -1), (2, 2), (-\sqrt{2}, 1), (-1, 2), (-2, 2) \right\} \checkmark$$

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$$2g^2(x)+1 = \left\{ (-2, 19), (1, 3), (3, 9), (-1, 1) \right\} \checkmark$$

$$\frac{2f}{g} = \left\{ (1, -4), (3, -1) \right\} \checkmark$$