۱۹ صیرفی ... آرا

$f(x) = \frac{1}{2}^{x-2} \Rightarrow f(x+1) = \left(\frac{1}{2}\right)^{x-1} - 1$ -۱

$g = \sqrt{x^3 f(x+1)} \Rightarrow x^3 f(x+1) \geq 0 \Rightarrow x^3 \times (2^{1-x} - 1) \geq 0$ ۲

$0 \quad 1$; $- \quad + \quad -$

$D_f = [0, 1]$ ✓

$f(x) = \sqrt{x + |x+2|} \Rightarrow f(-x) = \sqrt{-x + |2-x|}$ if $x \geq 2 = -x + x - 2 = -2$ زیر رادیکال باید نامنفی باشد ولی منفی است ۲ ۲

$x \leq 2 = -2x + 2$

$-2x + 2 \geq 0 \Rightarrow 2 \geq 2x$

$D_{f(-x)} = 1 \geq x$ $\quad 1 \geq x$ ✓

$\sqrt{\frac{x-1}{f(x)}}$: $\frac{x-1}{f(x)} \geq 0$ ریشه $x = 1$ ۲ -۳

$x = 2$, $x = 3$, $x = -4$

$-4 \quad 1 \quad 2 \quad 3$; $- \quad + \quad - \quad + \quad -$

$D(f) = (-4, 1] \cup (2, 3)$ ✓

$9x^4 - 7x^2 - 4x - 3 \neq 0 \Rightarrow (9x^4 - 6x^2 + 1) - (x^2 + 4x + 4) = (3x^2 - 1)^2 - (x+2)^2$ -۴ ۲

$\Rightarrow (3x^2 + x + 1)(3x^2 - x - 3) \Rightarrow D_f = \mathbb{R} - \{\text{ریشه‌های } 3x^2 - x - 3\}$, $D_g = \mathbb{R} - \{3x^2 + ax + b \text{ ریشه}\}$

$\Delta < 0$ ریشه ندارد

$3x^2 - x - 3 = 3x^2 + ax + b \rightarrow a = -1,\ b = -3$ ✓ $\quad a + b = -4 \Rightarrow (a \cdot b) = 3$

$\Rightarrow \sqrt{(a+b)} = \sqrt{4} = 2$ ✓

$f(x) = x^3 + x$

$f\left(\frac{4}{x}\right) = \left(\frac{4}{x}\right)^3 + \frac{4}{x} = \frac{64}{x^3} + \frac{4}{x}$ $\Big\}\ f(x) - f\left(\frac{4}{x}\right) = x^3 + x - \frac{64}{x^3} - \frac{4}{x}$ -۵

۱,۵

$g = \sqrt{f(x) - f\left(\frac{4}{x}\right)} \Rightarrow f(x) - f\left(\frac{4}{x}\right) \geq 0$

$\Rightarrow \left(x^3 - \frac{64}{x^3}\right) + \left(x - \frac{4}{x}\right) \geq 0 \Rightarrow \left(x - \frac{4}{x}\right)\left(x^2 + 4 + \frac{16}{x^2}\right) + \left(x - \frac{4}{x}\right) \geq 0$

$\frac{x^2 - 4}{x} \rightarrow$ $-2 \quad 0 \quad 2$; $- \quad + \quad - \quad +$

$\Rightarrow \left(x - \frac{4}{x}\right)\left(x^2 + 5 + \frac{16}{x^2}\right) \geq 0$ $\quad -2 \quad +2$; $+ \quad - \quad +$ $\quad D_g = \mathbb{R} - (-2, 2)$

$x = \frac{4}{x} \quad x^2 = 4 \quad x = \pm 2$ $\quad x^4 + 5x^2 + 16\ \ \Delta < 0$ همواره مثبت

سیده‌ای ۲۰ ـ ۶

(۲)

$f(x^3+x) = 2x^2 - 1$ $\Rightarrow f(1^3+1) = \frac{2-1}{1} = f(2) = 1$

$f(2)+f(10) = 8$ ✓ $\Rightarrow f(2^3+2) = 2\times 2^2 - 1 = f(10) = 7$

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(۲) ـ ۷

$f(x) = x^3 - 6x^2 + 12x$ $\quad x^3 - 3\times 2x^2 + 3\times 2^2 x - 2^3$ $\quad \cancel{x(x^2-6x+12)} = (x-2)^3 + 8$

$g(x) = x^3 + 9x^2 + 27x = (x+3)^3 - 27 = x^3 + 9\times x^2 + 3\times 3^2 \times x + 3^3 = x^3 + 9x^2 + 27x$

$f(x) = (x-2)^3 + 8 \quad f(\sqrt[3]{2}+2) = (\sqrt[3]{2})^3 + 8 = 10$

$g(x) = (x+3)^3 - 27 \quad g(\sqrt[3]{7} - 3) = \frac{(\sqrt[3]{7}-3+3)^3 - 27}{7 - 27 = -20}$ ✓

$\frac{-20}{10} = -2$

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(۲) ـ ۸

$f(x) = \sqrt{x + 4\sqrt{x-4}} = \sqrt{x-4+4\sqrt{x-4}+4} = \sqrt{(\sqrt{x-4}+2)^2} = |\sqrt{x-4}+2|$

$= \sqrt{x-4}+2 \quad g(x) = \sqrt{x - 4\sqrt{x-4}} = \sqrt{x-4-4\sqrt{x-4}+4} = \sqrt{(\sqrt{x-4}-2)^2} = |\sqrt{x-4}-2|$

$\Rightarrow |\sqrt{x-4}-2| \Rightarrow \begin{cases} \sqrt{x-4}-2 & x \geq 8 \\ -\sqrt{x-4}+2 & 4 \leq x \leq 8 \end{cases} \Rightarrow f(x)+g(x) = \begin{cases} a + b\sqrt{x-4} \\ 4 + 0\sqrt{x-4} \end{cases}$ ✓

$x \geq 8 \Rightarrow \frac{a+b}{c} = \frac{0+2}{-4} = -\frac{1}{2}$

این حالت کلا غلطه نامعلومه c ← $4 \leq x < 8 \Rightarrow \frac{a+b}{c} = \frac{4+0}{-4} = -1$

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(۲) ـ ۹

$g(x) = \{(2,1)\ (1,0)\ (3,5)\ (0,4)\}$

$f(x) = \frac{x+2}{x^2-4x+3} \Rightarrow f(x) = \{(2,-4), (1, \text{تن}), (3, \text{تن}), (0, \frac{2}{3})\}$

$(x-3)(x-1)$

$\frac{g}{f} = \{(2, -\frac{1}{4}), (0, 6)\}$ ✓

(۱٫۵) ✓ ـ ۱۰

$f(2x) = \{(\frac{1}{2}, 2), (\frac{3}{2}, -1), (2,2), (-\frac{1}{2}, 6)\}$

$f(x^2)\ \{(1,2), (\sqrt{3}, -1), (2,2)\}$ ریشه دوم!

$(-1,2)\quad (-\sqrt{3}, -1)\quad (-2,2)$

$2g^2(x)+1 = (-2, 19), (1, 3), (3, 9), (-1, 1)$ ✓

و) $\frac{4f}{g}\ \{(1,-4), (3,-1), \cancel{(-1)}\}$ ✓