

سوال ۱ -

$$f(x) = \left(\frac{1}{x}\right)^{x-2} - 1 \quad y = \sqrt{x^3 f(x+1)}$$

$$\left(\frac{1}{x}\right)^{(x+1)-2} - 1 = 0 \quad 2 - (1+x) = 1 \Rightarrow 2 - 1 - x = 0 \quad x = 1$$

جواب کنیم به ازای  $x$  بنا بر آنکه از  $x$  به بالا

$$\left(\frac{1}{x}\right)^{2-2} - 1 = -1 < 0$$

|     |   |   |
|-----|---|---|
| $x$ | 0 | 1 |
| $y$ | - | + |

$D_f = [0, 1]$

سوال ۲ -

$$f(x) = \sqrt{x + |x+2|} \quad f(-x) = ?$$

$$f(-x) = \sqrt{-x + |-x+2|} \Rightarrow \begin{cases} x > 2; & -x + (x-2) = -2 \quad \times \\ x \leq 2; & -x + (-x+2) = -2x+2 \quad \checkmark \end{cases}$$

$$\Rightarrow D_f = (-\infty, 2]$$

سوال ۳ -

$$y = \sqrt{\frac{x-1}{f(x)}} \quad \begin{array}{c|ccc} x & -2 & 1 & 2 & 3 \\ \hline y & - & + & - & + \end{array} \quad D_f = (-\infty, 1] \cup (2, 3)$$

$$y_1 = \frac{1}{4x^k - 5x^k - 6x^k - k} \Rightarrow 4x^k - 5x^k - 6x^k - k = 4x^k - 9x^k - k = -5x^k - k$$

$$y_2 = \frac{1}{k^2 x^2 + ax + b}$$

$$a = -1 \quad b = -2$$

$$= (k^2 x^2 - 1)^2 - (x+2)^2 = (k^2 x^2 - 1 - x - 2)(k^2 x^2 - 1 + x + 2)$$

$$= (k^2 x^2 - x - 3)(k^2 x^2 + x + 1)$$

$$\Delta = (1)^2 - 4(1)(3) < 0$$

$$\Rightarrow \sqrt{-(a+b)} = \sqrt{-(-1+(-2))} = \sqrt{-(-3)} = \sqrt{3} = 2$$

سوال ۵ -

$$y = \sqrt{f(x) - f\left(\frac{x}{2}\right)} \quad f(x) = x^k + x$$

$$y = \sqrt{(x^k + x) - \left(\frac{x^k}{2} + \frac{x}{2}\right)} = \sqrt{\frac{x^k + x - \frac{x^k}{2} - \frac{x}{2}}{1}} = \sqrt{\frac{\frac{x^k}{2} + \frac{x}{2}}{1}} = \sqrt{\frac{x^k + x}{2}}$$

$$= \sqrt{\frac{(x^k - 2)(x^k + 1) + x^k(x^k - 2)}{2}} = \sqrt{\frac{(x^k - 2)(x^k + x + 1)}{2}}$$

$$\Delta = (1)^2 - 4(1)(1) = -3 < 0$$

$$\Rightarrow \sqrt{-(a+b)} = \sqrt{-(-1+(-2))} = \sqrt{-(-3)} = \sqrt{3} = 2$$

$$\begin{array}{c|ccc} x & -2 & 0 & 2 \\ \hline y & - & + & - \end{array} \quad D_f = [-2, 0) \cup [2, +\infty)$$

سوال ۶ -

$$f(n^r + n) = r n^r - 1 \quad f(r) + f(1) = ?$$

$$\begin{aligned} \text{نکته:} \quad & n=1 \quad (1)^r + (1) = r \\ & n=r \quad (r)^r + (r) = 1. \end{aligned} \Rightarrow f(r) + f(1) = r(1)^r - 1 + r(r)^r - 1 = r - 1 + 1 - 1 = 1$$

سوال ۷ -

$$f(n) = n^r - 9n^r + 12n \quad g(\sqrt{r} - r)$$

$$g(n) = n^r + 9n^r + 2\sqrt{n} \quad f(\sqrt{r} + r)$$

$$\begin{aligned} n^r - 9n^r + 12n - 1 &= (n-r)^r \Rightarrow f(n) = (n-r)^r + 1 \\ n^r + 9n^r + 2\sqrt{n} + 2\sqrt{r} &= (n+r)^r \Rightarrow g(n) = (n+r)^r - 2\sqrt{n} \end{aligned}$$

$$\frac{g(\sqrt{r} - r)}{f(\sqrt{r} + r)} = \frac{((\sqrt{r} - r)^r - 2\sqrt{r})}{((\sqrt{r} + r)^r - 2\sqrt{r})} = \frac{r - 2\sqrt{r}}{r + 1}$$

$$= \frac{-10}{10} = -1$$

سوال ۸ -  
 $x > r$

$$f(n) = \sqrt{n + r\sqrt{n-r}} = \sqrt{(n-r) + r\sqrt{n-r} + r} = \sqrt{(\sqrt{n-r} + r)^2} = (\sqrt{n-r} + r)$$

$$g(n) = \sqrt{n - r\sqrt{n-r}} = \sqrt{(n-r) - r\sqrt{n-r} + r} = \sqrt{(\sqrt{n-r} - r)^2} = (\sqrt{n-r} - r)$$

$$f(n) + g(n) = a + b\sqrt{n+c} = (\sqrt{n-r} + r) + (\sqrt{n-r} - r) = 2\sqrt{n-r}$$

$$a=0 \quad b=1 \quad c=-r \Rightarrow \frac{a+b}{c} = \frac{0+1}{-r} = -\frac{1}{r}$$

$$f(n) = \frac{n+r}{n^r - r n + r} \quad g(n) = \{(r, 1), (1, 0), (r, r), (0, r)\} \quad \frac{g}{f} = ? \quad \text{سوال ۹ -}$$

$$\Rightarrow f(r) = \frac{(r)+r}{(r)^r - r(r) + r} = -r \quad f(1) = \frac{(1)+r}{(1)^r - r(1) + r} = \frac{r}{0} = \text{تعریف نشده}$$

$$f(r) = \frac{(r)+r}{(r)^r - r(r) + r} = \frac{0}{0} = \text{تعریف نشده} \quad f(0) = \frac{(0)+r}{(0)^r - r(0) + r} = \frac{r}{r}$$

$$\frac{g}{f} = \left\{ \left(r, -\frac{1}{r}\right), \left(0, \frac{r}{r}\right) \right\} \quad \frac{g}{f} = \left\{ \left(r, -\frac{1}{r}\right), \left(0, 1\right) \right\}$$

۱۰۰

$$f(x) = \{(1, 2), (2, -1), (3, 2), (-1, 6)\}$$

$$g(x) = \{(-2, 3), (1, -1), (3, 2), (-1, 0)\}$$

$$الف) f(2x) = \left\{ \left( \frac{1}{2}, 2 \right), \left( \frac{2}{2}, -1 \right), \left( \frac{3}{2}, 2 \right), \left( \frac{-1}{2}, 6 \right) \right\}$$

$$ب) f(x^2) = \{(1, 2), (\sqrt{3}, -1), (-\sqrt{3}, -1), (2, 2), (-2, 2)\}$$

$$ج) 2g^2(x) + 1 = \{(-2, 19), (1, 3), (3, 9), (-1, 1)\}$$

$$د) \frac{f}{g} = \left\{ \left( 1, \frac{2}{-1} \right), \left( 3, \frac{2}{2} \right) \right\}$$

نقطه شش  
چون بی معنی  
۱- را هم ندیم  
معنا ندارد

$$x = -1 \text{ تعریف نشده} \Rightarrow \frac{f(-1)}{g(-1)} = \frac{f(6)}{0}$$