

14, 5

تکلیف شماره 4

نام و نام خانوادگی: ابوالفضل ابوفاضلی

الف) $y = x^3 - 3x^2 + 3x = x(x^2 - 3x + 3) \Rightarrow x^2 - 3x + 3 : \begin{cases} a > 0 \\ \Delta = 9 - 12 < 0 \end{cases}$ (2)

$\Rightarrow x = \frac{y}{x^2 - 3x + 3} \rightarrow x^2 - 3x + 3 \neq 0 \Rightarrow D_y = \mathbb{R} \Rightarrow x = \frac{y}{x^2 - 3x + 3} \Rightarrow R_y = \mathbb{R}$ ✓

ب) $y = \frac{1}{x^2 - 2x} \Rightarrow yx^2 - 2yx - 1 = 0 \Rightarrow x = \frac{2y \pm \sqrt{4y^2 + 4y}}{2y} \Rightarrow \begin{cases} y \neq 0 \\ 4y^2 + 4y \geq 0 \end{cases}$

$\Rightarrow 4y(y+1) \geq 0 \Rightarrow \frac{-1}{+} \frac{0}{-} \frac{+}{+} \Rightarrow R_y = (-\infty, -1] \cup (0, +\infty)$ ✓

الف) $x^2 - 4x + 1 = 0 \rightarrow \text{ext. } \begin{cases} -\frac{b}{2a} = \frac{2}{1} = 2 \\ \frac{a}{c} = \frac{1}{1} = 1 \end{cases} \xrightarrow{a > 0} R_y = [2, +\infty)$ ✓ (2)

ب) $-x^2 + 4x + 3 = 0 \rightarrow \text{ext. } \begin{cases} -\frac{b}{2a} = \frac{-4}{-2} = 2 \\ \frac{a}{c} = \frac{-1}{3} = -\frac{1}{3} \end{cases} \xrightarrow{a < 0} (-\infty, 12] \Rightarrow R_y = (-\infty, 12]$ ✓

ج) $\sqrt{x^2 - 4x + 3} \rightarrow \text{ext. } \begin{cases} \frac{a}{c} = \frac{1}{3} = \frac{1}{3} \\ \frac{b}{2a} = \frac{-4}{2} = -2 \end{cases} \xrightarrow{a > 0} [-1, +\infty) \Rightarrow \sqrt{[-1, +\infty)} = [0, +\infty)$ ✓

د) $\sqrt{4x - x^2} \rightarrow \text{ext. } \begin{cases} -\frac{b}{2a} = \frac{-4}{-2} = 2 \\ \frac{a}{c} = \frac{-1}{4} = -\frac{1}{4} \end{cases} \xrightarrow{a < 0} (-\infty, 9] \Rightarrow R_y = [0, 3]$ ✓

الف) $y = x^3 - 6x^2 + 9x + 1 \xrightarrow{\text{بزرگترین توان: } x^3} R_y = \mathbb{R}$ ✓

ب) $y = x^4 - 4x^3 + 3x^2 + 2 \xrightarrow{\text{بزرگترین توان: } x^4} R_y = \mathbb{R}$ ✓

ج) $y = \sqrt{x^4 - 4x^3 + 6x^2 + 1} \Rightarrow R_y = \sqrt{\mathbb{R}} = [0, +\infty)$ ✓

د) $y = (x^4 - 4x^3 + 6x^2 + 1)^2 \Rightarrow R_y = (\mathbb{R})^2 = \mathbb{R} \quad [0, +\infty)$

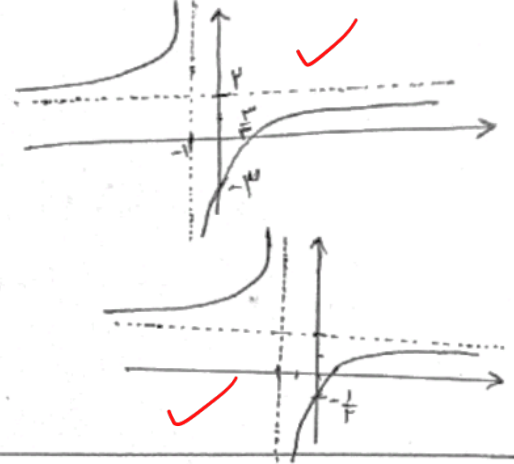
الف) $y = \frac{3x+1}{x-2} \Rightarrow R_y = \mathbb{R} - \left\{\frac{a}{c}\right\} \Rightarrow R_y = \mathbb{R} - \{3\}$ ✓ (2)

ب) $y = \frac{2x+1}{x+1} \xrightarrow{\frac{ad \neq bc}{c \neq 0}} R_y = \mathbb{R} - \left\{\frac{a}{c}\right\} \Rightarrow R_y = \mathbb{R} - \{2\}$ ✓

الف) $y = \sqrt{\frac{2x+1}{x+1}} \xrightarrow{\frac{ad \neq bc}{c \neq 0}} R_y = \mathbb{R} - \left\{\frac{a}{c}\right\} \Rightarrow R_y = \sqrt{\mathbb{R} - \{2\}} \Rightarrow R_y = [0, +\infty) - \{\sqrt{2}\}$ ✓ (2)

ب) $y = \sqrt{\frac{3x+1}{2-x}} \xrightarrow{\frac{ad \neq bc}{c \neq 0}} R_y = \mathbb{R} - \left\{\frac{a}{c}\right\} \Rightarrow R_y = \sqrt{\mathbb{R} - \{-2\}} \Rightarrow R_y = [0, +\infty)$ ✓

الف) $y = \frac{2x-3}{x+1}$ $\xrightarrow{ad \neq bc, c \neq 0}$ جانب $\left\{ \begin{array}{l} \text{افترق: } \frac{a}{c} = 2 \\ \text{تعرى: } x+1 \neq 0 \rightarrow x \neq -1 \end{array} \right.$



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ب) $y = \frac{2x-1}{x+2}$ $\xrightarrow{ad \neq bc, c \neq 0}$ جانب $\left\{ \begin{array}{l} \text{افترق: } \frac{a}{c} = 2 \\ \text{تعرى: } x+2 \neq 0 \rightarrow x \neq -2 \end{array} \right.$

الف) $y = \sin x + \frac{1}{\sin x}$ $\xrightarrow{-1 \leq \sin x \leq 1}$ $Ry = (-\infty, -2] \cup [2, +\infty)$ $\checkmark$ $\sin x = 1 \checkmark$

2

ب) $\frac{x^4+1}{x^3} = x + \frac{1}{x^3}$ $\xrightarrow{\frac{x^4}{x^3} < 0, \frac{x^4}{x^3} > 0}$ $Ry = (-\infty, -2] \cup [2, +\infty)$ $\checkmark$ $x^3 = 1 \checkmark$

✓

ج) $\frac{\sqrt{x^2+1}}{\sqrt{x}} = \sqrt{x} + \frac{1}{\sqrt{x}}$ $\xrightarrow{\sqrt{x} < 0, \sqrt{x} > 0}$ $Ry = (-\infty, -2] \cup [2, +\infty)$ $\checkmark$

د) $\sqrt{x} + \frac{1}{\sqrt{x}}$ $\xrightarrow{\sqrt{x} > 0, \sqrt{x} = 1}$ $Ry = [2, +\infty)$ $\checkmark$

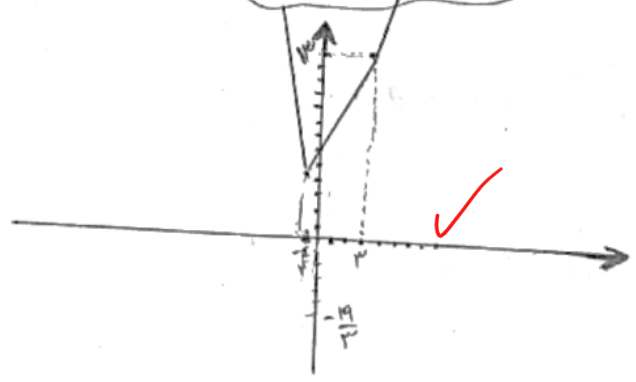
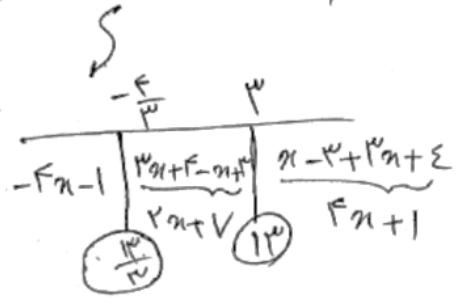
الف) $x^2 + \frac{1}{x^2+3}$ $\rightarrow \left(x^2+3 + \frac{1}{x^2+3} \right) - 3 \Rightarrow Rf = [x^2+3, +\infty) \Rightarrow Ry = [x^2+3, -\infty) - 3$
 $\Rightarrow Ry = [2, +\infty)$ $\checkmark$

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ب) $\frac{x^2+8}{\sqrt{x^2+4}} = \sqrt{x^2+4} + \frac{1}{\sqrt{x^2+4}}$ $\xrightarrow{\frac{x^2+8}{\sqrt{x^2+4}} \neq 1, \frac{x^2+8}{\sqrt{x^2+4}} > 0}$ $\min_{x^2+4} = 2 \Rightarrow Ry = [2+\frac{1}{2}, +\infty)$
 $\Rightarrow Ry = [2.5, +\infty)$ $\checkmark$

$y = |x-3| + |3x+5|$



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الف) $y = |x-1| + |2x+2| \rightarrow \begin{cases} 3x, x \geq 1 \rightarrow R = [3, +\infty) \\ x+4, -1 \leq x \leq 1 \rightarrow R = [4, 4] \\ -2x, x \leq -1 \rightarrow R = [2, +\infty) \end{cases} \cup Ry = [3, +\infty)$ $\checkmark$

2

ب) $y = |2x-2| - |x+1| \rightarrow \begin{cases} x-3, x \geq 1 \rightarrow R = [-2, +\infty) \\ -3x+1, -1 \leq x \leq 1 \rightarrow R = [-2, 2] \\ -x+3, x \leq -1 \rightarrow R = [4, +\infty) \end{cases} \cup Ry = [-2, +\infty)$ $\checkmark$

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