

A, 1, 15

الف) $x^3 - 3x^2 + 3x = (x-1)^3 + 1 = y \Rightarrow x-1 = \sqrt[3]{y-1} \Rightarrow x = \sqrt[3]{y-1} + 1$
 $R_f = \mathbb{R} \checkmark$

(۲)

ب) $x^2 - 2x = (x-1)^2 - 1 \Rightarrow y = (x-1)^2 - 1 \Rightarrow y+1 = (x-1)^2 \Rightarrow x-1 = \pm \sqrt{y+1} \Rightarrow x = 1 \pm \sqrt{y+1}$
 $\Rightarrow x-1 = \sqrt{\frac{y+1}{y}} \Rightarrow x = \sqrt{\frac{y+1}{y}} + 1 \Rightarrow R_f: (-\infty, -1] \cup (0, +\infty) \checkmark$

الف) $-\frac{b}{a} = \frac{4}{2} = 2 \Rightarrow y_{min} = 2^2 - 2 \times 2 + 1 = 1 \Rightarrow R_f: [1, +\infty) \checkmark$

(۲)

ب) $-\frac{b}{a} = \frac{4}{2} = 2 \Rightarrow y_{max} = -2^2 + 2 \times 2 + 1 = 1 \Rightarrow R_f: (-\infty, 1] \checkmark$

ج) $-\frac{b}{a} = 2 \Rightarrow y_{min} = 2^2 - 4 \times 2 - 3 = -11 \Rightarrow R_f: \sqrt{-11, +\infty) = [0, +\infty) \checkmark$

د) $-\frac{b}{a} = 3 \Rightarrow y_{max} = 11 - 9 = 2 \Rightarrow R_f: [0, 2] \checkmark$

الف) $R_f: \mathbb{R} \checkmark$

ب) $R_f: \mathbb{R} \checkmark$

د) $R_f: (\mathbb{R})^2 = [0, +\infty) \checkmark$

(۲)

ج) $R_f: \sqrt{\mathbb{R}} = [0, +\infty) \checkmark$

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الف) $y = 4x - 2y = 3x + 1 \Rightarrow y = 4x - 3x + 1 \Rightarrow y = x + 1 \Rightarrow x = y - 1$
 $\Rightarrow R_f: \mathbb{R} - \{1\} \checkmark$

(۲)

ب) $R_f: \mathbb{R} - \{\frac{9}{2}\} = \mathbb{R} - \{4.5\} \checkmark$

$4x + 1 = 2x + 1 \Rightarrow x = 0 \Rightarrow x = \frac{0}{4-2} \Rightarrow \mathbb{R} - \{2\}$

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الف) $\mathbb{R} - \{\frac{9}{2}\} \Rightarrow R_f: \sqrt{\mathbb{R} - \{2\}} = [0, +\infty) - \{\sqrt{2}\} \checkmark$

(1, 15)

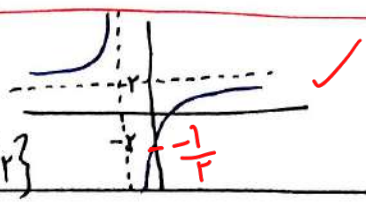
ب) $\mathbb{R} - \{\frac{9}{2}\} \Rightarrow R_f: \sqrt{\mathbb{R} - \{2\}} = [0, +\infty) - \{\sqrt{2}\} \checkmark$
 $\frac{3x+1}{-x+2} = -3 \Rightarrow \frac{a}{c} = -3$

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(الف) محل برخورد با محورها $x+1=0 \Rightarrow x=-1$ مجانِب قائم: $y=2$
مجانِب افقی: $y=\frac{a}{c} = y=2$ $R_f: \mathbb{R}-\{2\}$



(ب) مجانِب قائم: $x=-2$
مجانِب افقی: $y=2$ $R_f: \mathbb{R}-\{2\}$



(الف) $\sin x + \frac{1}{\sin x}$ $\sin x > 0 \rightarrow R_y: [2, +\infty)$
 $\sin x < 0 \rightarrow R_y: (-\infty, -2]$ $R_y = (-\infty, -2] \cup [2, +\infty)$

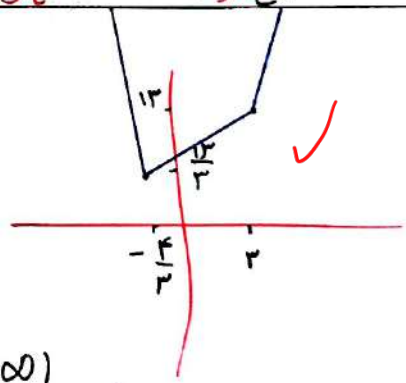
(ب) $x^3 + \frac{1}{x^3}$ $x^3 > 0 \rightarrow R_y: [2, +\infty)$
 $x^3 < 0 \rightarrow R_y: (-\infty, -2]$ $R_y: (-\infty, -2] \cup [2, +\infty)$

(ج) $\sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}$ $\sqrt[3]{x} > 0 \rightarrow R_y: [2, +\infty)$
 $\sqrt[3]{x} < 0 \rightarrow R_y: (-\infty, -2]$ $R_y: (-\infty, -2] \cup [2, +\infty)$ (د) $\sqrt{x} > 0 \Rightarrow R_y = [2, +\infty)$
غیر قابل

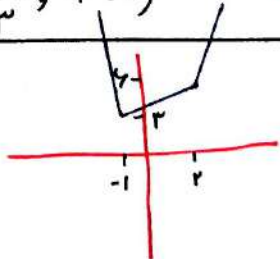
کمترین مقدار x^2 مفراست پس بجای x^3 صفر قرار می دهیم تا y_{min} بدست بیاد
 $x^2 \dots \rightarrow \dots + \frac{1}{0+x^3} = \frac{1}{x^3} = y_{min} \Rightarrow R_f: [\frac{1}{3}, +\infty)$

(ب) $\frac{x^2+4+1}{\sqrt{x^2+4}} = \sqrt{x^2+4} + \frac{1}{\sqrt{x^2+4}}$ $x > 0 \rightarrow 2 + \frac{1}{x}, 2.5$
 $x < 0$ غیری $R_f: [2.5, +\infty)$

$x > 3 \rightarrow 4x+1$
 $-\frac{4}{3} \leq x \leq 3 \rightarrow 2x+7 \rightarrow \left[\frac{10}{3}, 13 \right]$
 $x < -\frac{4}{3} \rightarrow -4x-1$ $R_f: [\frac{13}{3}, +\infty)$



(الف) $x > 2 \rightarrow 3x$
 $-1 \leq x \leq 2 \rightarrow x+4 \rightarrow [3, 6]$
 $x < -1 \rightarrow -2x$



(ب) $x > 1 \rightarrow x-3$
 $-1 \leq x \leq 1 \rightarrow -2x+1 \rightarrow [-1, 3]$
 $x < -1 \rightarrow -x+3$

