

الف) $y = x^3 - 3x^2 + 3x \Rightarrow y+1 = x^3 - 3x^2 + 3x + 1$

$\Rightarrow (x-1)^3 = y+1 \Rightarrow x-1 = \sqrt[3]{y+1} \Rightarrow x = \sqrt[3]{y+1} + 1 \Rightarrow R_y = \mathbb{R}$

ب) $y = \frac{1}{x^2 - 2x} \Rightarrow x^2 y - 2xy = 1 \Rightarrow x^2 - 2x = \frac{1}{y} \Rightarrow x^2 - 2x = \frac{1}{y} + 1$

$\Rightarrow (x-1)^2 = \frac{y+1}{y} \Rightarrow x-1 = \sqrt{\frac{y+1}{y}} \Rightarrow x = \sqrt{\frac{y+1}{y}} + 1 \Rightarrow \frac{y+1}{y} \geq 0$
 $\Rightarrow R_y = (-\infty, -1) \cup (0, +\infty)$

الف) $y = x^2 - 4x + 10 \quad \text{ext} \left| \begin{smallmatrix} -\frac{4}{2} = 2 \\ a > 0 \end{smallmatrix} \right\} \Rightarrow R_y = [6, +\infty)$

ب) $y = -x^2 + 6x + 7 \quad \text{ext} \left| \begin{smallmatrix} \frac{6}{2} = 3 \\ a < 0 \end{smallmatrix} \right\} \Rightarrow R_y = (-\infty, 12]$

ج) $y = \sqrt{x^2 - 4x - 7} \quad \text{ext} \left| \begin{smallmatrix} \frac{4}{2} = 2 \\ a > 0 \end{smallmatrix} \right\} \Rightarrow R_y = [0, +\infty)$

د) $y = \sqrt{4x - x^2} \Rightarrow y = \sqrt{-x^2 + 4x} \quad \text{ext} \left| \begin{smallmatrix} \frac{4}{2} = 2 \\ a < 0 \end{smallmatrix} \right\} \Rightarrow R_y = [0, 2]$

الف) $y = x^2 - 5x^2 + 2x + 1 \Rightarrow R_y = \mathbb{R}$

ب) $y = x^2 - 6x^2 + 3x + 2 \Rightarrow R_y = \mathbb{R}$

ج) $y = \sqrt{x^2 - 6x^2 + 5x + 1} \Rightarrow R_y = [0, +\infty)$

د) $y = (x^2 - 6x^2 + 5x + 1)^{\frac{1}{2}} \Rightarrow R_y = [0, +\infty)$

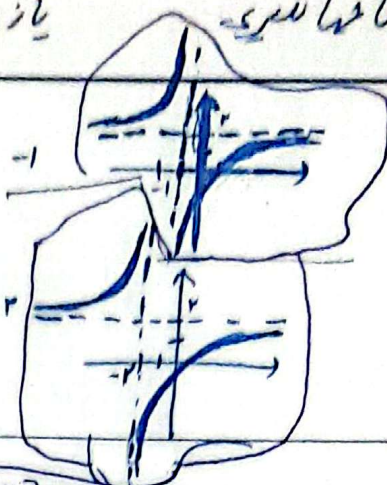
الف) $y = \frac{3x+1}{x-2} \quad \text{ad} \neq bc \Rightarrow R_y = \mathbb{R} - \{3\}$

ب) $y = \frac{3x+1}{x+1} \quad \text{ad} \neq bc \Rightarrow R_y = \mathbb{R} - \{3\}$

الف) $y = \sqrt{\frac{3x+1}{x+1}} \Rightarrow R_y = [0, +\infty) - \{3\}$

ب) $y = \sqrt{\frac{3x+1}{x-1}} \Rightarrow R_y = [0, +\infty)$

الف) $y = \frac{2x-3}{x+1} \rightarrow \begin{cases} \text{میانگین} = \frac{a}{c} = 2 \\ \text{نسبت} = \frac{b}{c} = -1 \end{cases}$
 $x=0 \Rightarrow y = -3$



ب) $y = \frac{2x-1}{x+2} \rightarrow \begin{cases} \text{میانگین} = \frac{a}{c} = 2 \\ \text{نسبت} = \frac{b}{c} = -2 \end{cases}$
 $x=0 \Rightarrow y = -\frac{1}{2}$

الف) $y = \frac{1}{\sin x} \xrightarrow{a+\frac{1}{a}} R_y = (-\infty, -1] \cup [1, +\infty)$

ب) $y = \frac{1}{x^2} \xrightarrow{a+\frac{1}{a}} R_y = (-\infty, -1] \cup [1, +\infty)$

ج) $y = \frac{1}{\sqrt{x}} \xrightarrow{a+\frac{1}{a}} R_y = (-\infty, -1] \cup [1, +\infty)$

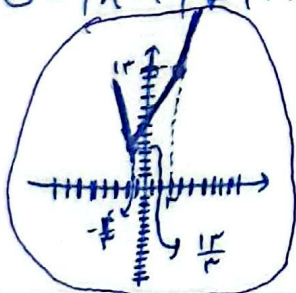
د) $y = \frac{1}{\sqrt{x}} \xrightarrow{a+\frac{1}{a}} R_y = [1, +\infty)$

الف) $y = x^2 + \frac{1}{x^2+3} \Rightarrow y = \left(x^2 + 3 + \frac{1}{x^2+3}\right) - 3$

$y = x^2 + 3 + \frac{1}{x^2+3} \xrightarrow{a+\frac{1}{a}} R_y = \left[3 + \frac{1}{3}, +\infty\right) = \left[\frac{10}{3}, +\infty\right)$

$\Rightarrow R_y = \left[\frac{1}{3}, +\infty\right)$
 ب) $y = \frac{x^2+1}{x^2+3} = \frac{(x^2+3)+1}{x^2+3} = 1 + \frac{1}{x^2+3} \xrightarrow{a+\frac{1}{a}} R_y = \left[1 + \frac{1}{3}, +\infty\right) = \left[\frac{4}{3}, +\infty\right)$

$y = |x-3| + (3x+5)$



$\begin{array}{c} -1 \qquad 4 \\ \hline -x+3+3x+5 \quad -x+3+3x+5 \quad x-3+3x+5 \\ = -x+8 \quad = 2x+8 \quad = 4x+2 \end{array}$

$R_y = \left[\frac{14}{4}, +\infty\right) \quad D_y = \mathbb{R}$

الف) $y = |x-2| + |2x+2|$

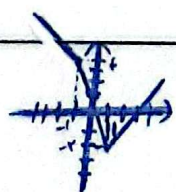


$\rightarrow R_y = [3, +\infty)$

$\begin{array}{c} -1 \qquad 3 \\ \hline -x+2+2x+2 \quad -x+2+2x+2 \quad x-2+2x+2 \\ = -x+4 \quad = x+4 \quad = 3x \end{array}$

ب) $y = |2x-2| - (x+1)$

$\begin{array}{c} -1 \qquad 1 \\ \hline -2x+2-x-1 \quad -2x+2-x-1 \quad 2x-2-x-1 \\ = -3x+1 \quad = -3x+1 \quad = x-3 \end{array}$



$\rightarrow R_y = [-1, +\infty)$