

$$a) y = x^2 - 2x^2 + 2x \Rightarrow y - 1 = x^2 - 2x^2 + 2x - 1 \Rightarrow y - 1 = (x-1)^2 \Rightarrow \sqrt{y-1} = x-1 \\ \Rightarrow x = \sqrt{y-1} + 1$$

$$b) y = \frac{1}{x^2 - 2x} \Rightarrow 1 = yx^2 - y2x \Rightarrow 1 = y(x^2 - 2x) \Rightarrow \frac{1}{y} = x^2 - 2x$$

$$\frac{1}{y} + 1 = x^2 - 2x + 1 \Rightarrow (x-1)^2 = \frac{1}{y} + 1 \Rightarrow \sqrt{\frac{1}{y} + 1} = x-1 \Rightarrow$$

$$x = \sqrt{\frac{1}{y} + 1} + 1 \Rightarrow \frac{1}{y} + 1 \geq 0 \Rightarrow \frac{y+1}{y} \geq 0 \Rightarrow \frac{-1}{y} \leq 1 \Rightarrow \frac{1}{y} \geq -1 \Rightarrow y \in (-\infty, -1] \cup (0, +\infty)$$

$$a) y = x^2 - 4x + 1 \Rightarrow \text{ext} \left\{ \begin{array}{l} \frac{b}{2a} = 2 \\ f(2) = -7 \end{array} \right. \Rightarrow R_y = [-7, +\infty)$$

$$b) y = -x^2 + 4x + 3 \Rightarrow \text{ext} \left\{ \begin{array}{l} \frac{b}{2a} = 2 \\ f(2) = 7 \end{array} \right. \Rightarrow R_y = (-\infty, 7]$$

$$c) y = \sqrt{x^2 - 4x - 3} \Rightarrow \text{ext} \left\{ \begin{array}{l} \frac{b}{2a} = 2 \\ f(2) = -5 \end{array} \right. \Rightarrow R(\sqrt{x^2 - 4x - 3}) = [-5, +\infty)$$

$$d) y = \sqrt{-x^2 + 4x} \Rightarrow \text{ext} \left\{ \begin{array}{l} \frac{b}{2a} = 2 \\ f(2) = 2 \end{array} \right. \Rightarrow R(\sqrt{-x^2 + 4x}) = [0, 2]$$

$$a) y = x^2 - 5x^2 + 2x + 1 \Rightarrow \text{پارابول تنگ} \Rightarrow R_y = \mathbb{R}$$

$$b) y = x^4 - 4x^2 + 2x + 2 \Rightarrow \text{پارابول تنگ} \Rightarrow R_y = \mathbb{R}$$

$$c) y = \sqrt{x^4 - 4x^2 + 2x + 1} \Rightarrow \text{پارابول تنگ} \Rightarrow R(\sqrt{x^4 - 4x^2 + 2x + 1}) = \mathbb{R}$$

$$d) y = (x^4 - 4x^2 + 2x + 1)^2 \Rightarrow \text{پارابول تنگ} \Rightarrow R(\sqrt{x^4 - 4x^2 + 2x + 1}) = [0, +\infty)$$

الف) $g = \frac{2n+1}{n-2} \quad ad \neq bc \quad IR - \frac{a}{c}$
 $\Rightarrow Rg = IR - \{2\}$

ب) $g = \frac{2n+1}{n+1} \quad ad \neq bc \quad Rg = IR - \frac{a}{c} = IR - \{2\}$

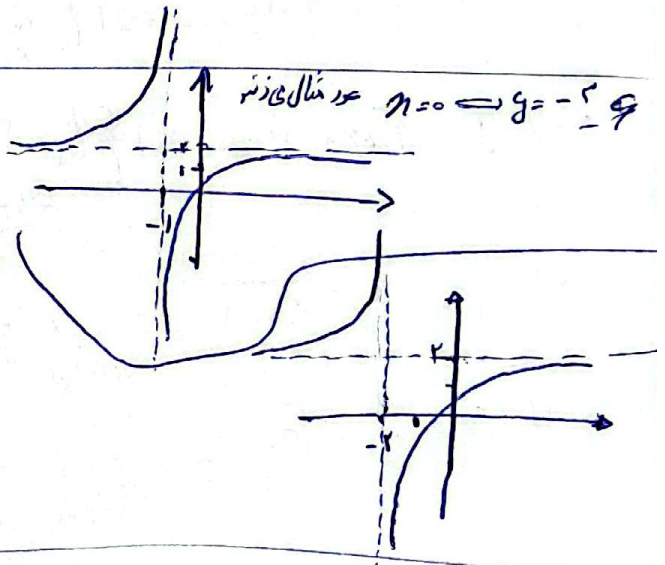
الف) $g = \sqrt{\frac{2n+1}{n+1}}$ $\rightarrow ad \neq bc \Rightarrow \dots \Rightarrow Rg = [0, +\infty)$

ب) $g = \sqrt{\frac{2n+1}{n+1}}$ $\rightarrow ad \neq bc \Rightarrow \dots \Rightarrow Rg = [0, +\infty)$

الف) $g = \frac{2n-2}{n+1}$ $\begin{cases} \text{رشته ی افقی} = \text{میانگین} \\ \text{میانگین} = \frac{a}{c} = 2 \end{cases}$

ب) $g = \frac{2n-1}{n+2}$ $\begin{cases} \text{رشته ی افقی} = \text{میانگین} \\ \text{میانگین} = \frac{a}{c} = 2 \end{cases}$

در مثال $n \rightarrow \infty \Rightarrow g = \frac{1}{2}$



الف) $g = \frac{\sin n}{\sin n} + \frac{1}{\sin n} = a + \frac{1}{a} = g \Rightarrow Rg = [2, +\infty) \cup (-\infty, -2]$

ب) $g = \frac{a^n}{a^n} + \frac{1}{a^n} = a + \frac{1}{a} = g \Rightarrow Rg = [2, +\infty) \cup (-\infty, -2]$

ج) $g = \frac{\sqrt{n}}{\sqrt{n}} + \frac{1}{\sqrt{n}} = a + \frac{1}{a} = g \Rightarrow Rg = [2, +\infty) \cup (-\infty, -2]$

د) $g = \frac{a}{\sqrt{n}} + \frac{1}{\sqrt{n}} = a + \frac{1}{a} = g \Rightarrow Rg = [2, +\infty)$

$$a) \quad g = a^r + \frac{1}{a^{r+r}} \Rightarrow g = \left(a^r + r + \frac{1}{a^{r+r}} \right) - r$$

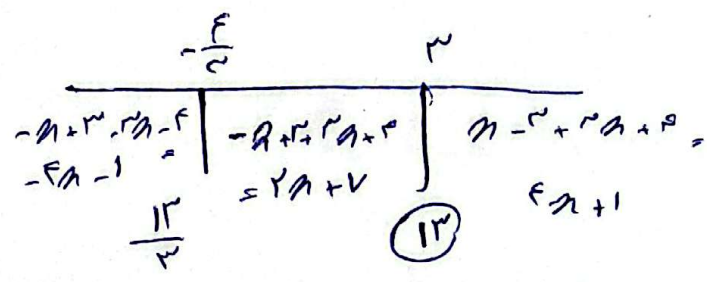
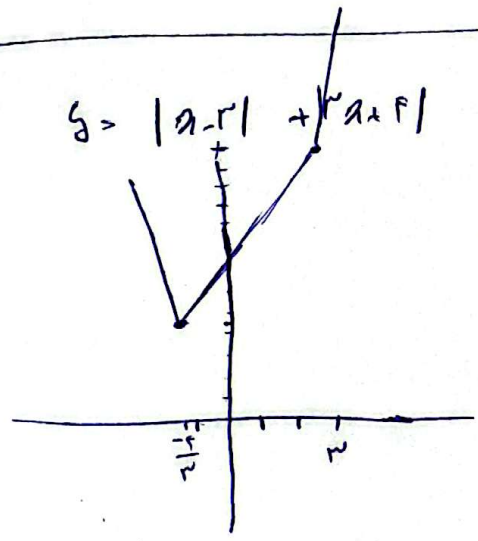
$$g_1 = a^r + r + \frac{1}{a^{r+r}} \xrightarrow{a > \frac{1}{a}} a^r + r \geq r$$

$$Pg_1 = \left[r + \frac{1}{r} + \infty \right) = \left[\frac{1}{r} + \infty \right) \Rightarrow Pg = \left[\frac{1}{r} + \infty \right)$$

$$b) \quad g = \frac{a^r + a}{\sqrt{a^{r+r}}} = \sqrt{a^r + r} + \frac{1}{\sqrt{a^{r+r}}} \xrightarrow{a > \frac{1}{a}} \sqrt{a^r + r} \geq r$$

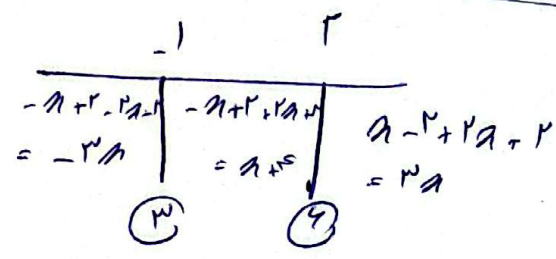
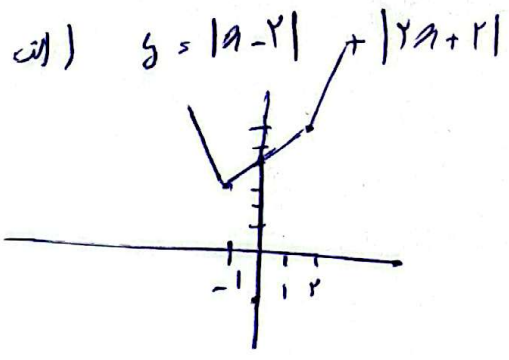
$$Pg = \left[r + \frac{1}{r} + \infty \right) =$$

$$Pg = \left[\frac{a}{r} + \infty \right)$$



$$Pg = \left[\frac{1}{r} + \infty \right) \quad Pg = \mathbb{R}$$

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