

سوال ۱)

الف) $y = x^3 - 3x^2 + 3x + 1$ $\rightarrow$ در توابع چند جمله ای مرتبه فرد (به انگیزه می توان فرض شد) به دنبال $\mathbb{R}$ است $R_f = \mathbb{R}$ راه اول:

$y = x^3 - 3x^2 + 3x + 1 = (x-1)^3 + 1 \Rightarrow y-1 = (x-1)^3 \Rightarrow x-1 = \sqrt[3]{y-1} \Rightarrow x = \sqrt[3]{y-1} + 1 \quad R_f = \mathbb{R}$

ب) $y = \frac{1}{x^2 - 2x}$

$\Rightarrow y = \frac{1}{x^2 - 2x + 1 - 1} = \frac{1}{(x-1)^2 - 1} \Rightarrow (x-1)^2 - 1 = \frac{1}{y} \Rightarrow (x-1)^2 = \frac{1}{y} + 1$

$\Rightarrow x-1 = \pm \sqrt{\frac{1}{y} + 1} \Rightarrow x = \pm \sqrt{\frac{1}{y} + 1} + 1$

$\frac{1}{y} + 1 > 0$
 $\frac{1}{y} > -1 \Rightarrow y < -1$
 $y \neq 0 \checkmark$

سوال ۲)

الف) $y = x^2 - 4x + 10$

$R_f = (-\infty, 1] \cup (6, +\infty)$

$a > 0 \rightarrow \text{Min}$

$\left\{ -\frac{b}{2a} = -\frac{(-4)}{2(1)} = 2 \Rightarrow (2)^2 - 4(2) + 10 = 6 \Rightarrow R_f = [6, +\infty) \right.$

ب) $y = -x^2 + 4x + 3$

$a < 0 \rightarrow \text{Max}$

$\left\{ -\frac{b}{2a} = -\frac{(-4)}{2(-1)} = 2 \Rightarrow -(2)^2 + 4(2) + 3 = 11 \Rightarrow R_f = (-\infty, 11] \right.$

ج) $y = \sqrt{x^2 - 4x - 3}$

$a > 0 \rightarrow \text{Min}$

$\left\{ -\frac{b}{2a} = -\frac{(-4)}{2(1)} = 2 \Rightarrow y = \sqrt{(2)^2 - 4(2) - 3} = \sqrt{4 - 8 - 3} = \sqrt{-7} \text{ تعریف نشده} \Rightarrow R_f = [0, +\infty) \right.$

د) $y = \sqrt{4x - x^2}$

$a < 0 \rightarrow \text{Max}$

$\left\{ -\frac{b}{2a} = -\frac{(4)}{2(-1)} = 2 \Rightarrow y = \sqrt{4(2) - (2)^2} = \sqrt{4 - 4} = 0 \quad R_f = \sqrt{[-\infty, 4]} \Rightarrow R_f = [0, 4] \right.$

الف) $y = x^2 - 5x^2 + 2x + 1$ $R_f = \mathbb{R}$

ب) $y = x^5 - 6x^2 + 2x + 2$ $R_f = \mathbb{R}$

ج) $y = \sqrt{x^5 - 6x^2 + 2x + 1}$ $R_f = \sqrt{\mathbb{R}} = [0, +\infty)$

د) $y = (x^5 - 6x^2 + 2x + 1)^2$ $R_f = \mathbb{R}^2 = [0, +\infty)$

الف) $y = \frac{2x+1}{x-2}$ $R_f = \mathbb{R} - \{\frac{1}{2}\}$

(سوال ۴)

ب) $y = \frac{2x+1}{x+1}$ $R_f = \mathbb{R} - \{2\}$

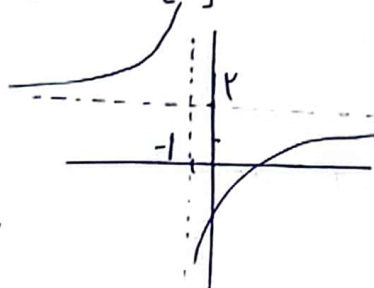
الف) $y = \sqrt{\frac{2x+2}{x+1}}$ $R_f = \sqrt{\mathbb{R} - \{2\}} = [0, +\infty) - \{\sqrt{2}\}$

(سوال ۵)

ب) $y = \sqrt{\frac{2x+1}{2-x}}$ $R_f = \sqrt{\mathbb{R} - \{-2\}} = [0, +\infty)$

الف) $y = \frac{2x-3}{x+1}$

(سوال ۶)



نقطه برخورد = -1 = جانب چپ

میانگین = $\frac{a}{c} = 2$

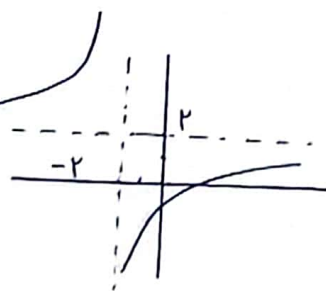
$$x=3 \Rightarrow \frac{2(3)-3}{(3)+1} = \frac{3}{4}$$

ب) $y = \frac{2x-1}{x+2}$

میانگین قائم = -2

میانگین افقی = $\frac{a}{c} = 2$

$$x=3 \Rightarrow \frac{2(3)-1}{(3)+2} = \frac{5}{5} = 1$$



$$الف) y = \frac{\sin x + 1}{\sin x} \quad R_f = (-\infty, -1] \cup [1, \infty)$$

$$\begin{cases} \sin x > 0 \rightarrow R_f = [1, \infty) \\ \sin x < 0 \rightarrow R_f = (-\infty, -1] \end{cases}$$

$$ب) y = \frac{x^2 + 1}{x^2} = x^2 + \frac{1}{x^2} \quad R_f = (-\infty, -1] \cup [1, \infty)$$

$$ج) y = \frac{\sqrt[3]{x^2 + 1}}{\sqrt[3]{x}} = \sqrt[3]{x} + \frac{1}{\sqrt[3]{x}} \quad R_f = (-\infty, -1] \cup [1, \infty)$$

$$د) y = \sqrt{x} + \frac{1}{\sqrt{x}} \Rightarrow R_f = [1, \infty)$$

$$الف) y = x^2 + \frac{1}{x^2 + 1} = x^2 + \frac{1}{x^2 + 1} + 1 - 1 = x^2 + 1 + \frac{1}{x^2 + 1} - 1$$

$$x^2 + 1 \xrightarrow{\text{min}} \Rightarrow -\frac{b}{2a} = -\frac{0}{2(1)} = 0 \Rightarrow \text{min} : 1 \Rightarrow 1 + \frac{1}{1} - 1 = 1$$

$$\Rightarrow R_f = [1, \infty)$$

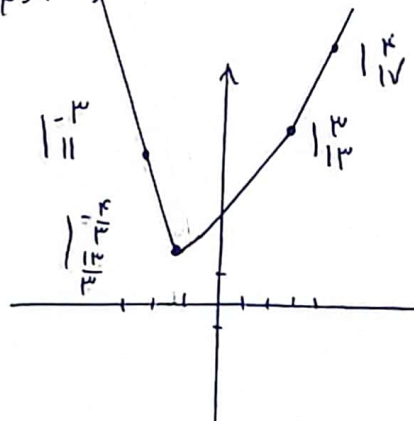
$$ب) y = \frac{x^2 + 1}{\sqrt{x^2 + 1}} = \frac{x^2 + 1}{\sqrt{x^2 + 1}} = \sqrt{x^2 + 1} + \frac{1}{\sqrt{x^2 + 1}} \quad x^2 + 1 \Rightarrow \text{min} = \frac{-b}{2a} = -\frac{0}{2(1)} = 0$$

$$y_{\text{min}} = (-1)^2 + 1 = 2$$

$$\Rightarrow y = \sqrt{1} + \frac{1}{\sqrt{1}} = 1 + 1 = 2 \Rightarrow R_f = [2, \infty)$$

$$y = |x - 1| + |x^2 + 1|$$

$$\begin{cases} x > 1; & x - 1 + x^2 + 1 = x^2 + x \\ -1 \leq x \leq 1; & 1 - x + x^2 + 1 = x^2 - x + 2 \\ x < -1; & 1 - x - x^2 - 1 = -x^2 - x \end{cases}$$



(9 د)

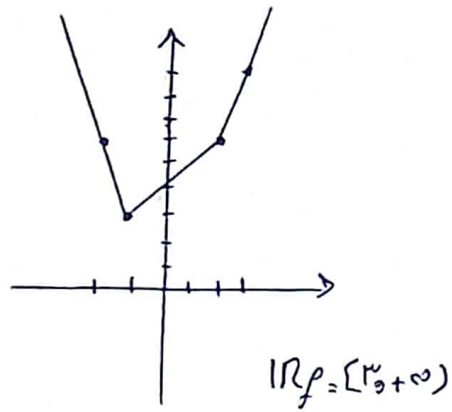
$$a) y = |x-2| + |x+2|$$

$$\Rightarrow y = |2-2| + |2+2| = 4$$

$$\Rightarrow y = |-1-2| + |-1+2| = 3$$

$$\Rightarrow y = |3-2| + |3+2| = 9$$

$$\Rightarrow y = |-3-2| + |-3+2| = 6$$



$$b) y = |x+1| - |x-1|$$

$$\Rightarrow y = |1+1| - |1-1| = 2$$

$$\Rightarrow y = |-1+1| - |-1-1| = -2$$

$$\Rightarrow y = |1+1| - |1-1| = 2$$

$$\Rightarrow y = |-1+1| - |-1-1| = -2$$

