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الف) $x^3 - 3x^2 + 3x = y \rightsquigarrow y = x^3 - 3x^2 + 3x - 1 + 1 \rightsquigarrow y = (x-1)^3 + 1 \rightsquigarrow \sqrt[3]{y-1} = x$

$\rightsquigarrow y \in \mathbb{R} \rightarrow R_f = \mathbb{R}$

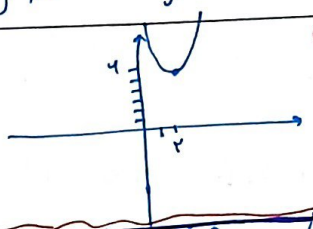
ب) $y = \frac{1}{x^2 - 2x} \rightsquigarrow yx^2 - 2yx = 1 \rightarrow yx^2 - 2yx - 1 = 0 \xrightarrow{\Delta} \frac{2y \pm \sqrt{4y^2 + 4y}}{2y}$

① $4y^2 + 4y \geq 0 \rightarrow y \in \mathbb{R}$ ② $2y \neq 0 \rightarrow y \neq 0$ $\left\{ \begin{array}{l} \text{① و ②} \\ \text{②} \end{array} \right\} R_f = \mathbb{R} - \{0\}$

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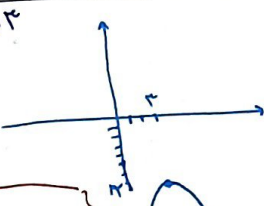
الف) $x^2 - 4x + 1 = y$

$\rightarrow R_f = [4 + \infty)$



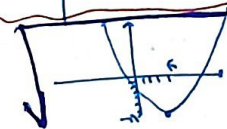
ب) $y = -x^2 + 4x + 3$

$\rightarrow R_f = (-\infty, 7]$



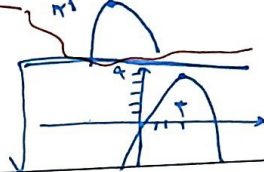
ج) $y = \sqrt{x^2 - 4x - 3}$

$\rightarrow R_f = [0 + \infty)$



د) $y = \sqrt{4x - x^2}$

$\rightarrow R_f = [0, 2]$



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الف) $y = x^3 - 3x^2 + 2x + 1 \rightsquigarrow R_f = \mathbb{R}$

ب) $y = x^3 - 4x^2 + 3x + 2 \rightsquigarrow R_f = \mathbb{R}$

ج) $y = \sqrt{x^3 - 4x^2 + 3x + 1} \rightsquigarrow R_f = [0 + \infty)$

د) $y = (x^3 - 4x^2 + 3x + 1)^2 \rightsquigarrow R_f = [0 + \infty)$

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الف) $y = \frac{x+1}{x-2}$

$\rightsquigarrow R_f = \mathbb{R} - \{\frac{1}{2}\} \rightsquigarrow R_f = \mathbb{R} - \{2\}$

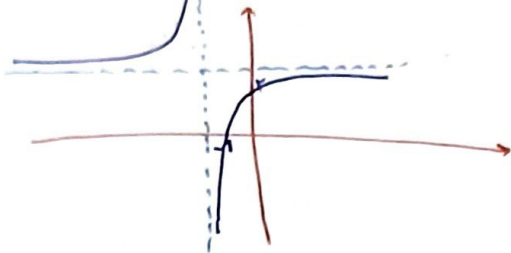
ب) $y = \frac{x+1}{x+1}$

$\rightsquigarrow R_f = \mathbb{R} - \{\frac{1}{2}\} \rightsquigarrow R_f = \mathbb{R} - \{1\}$

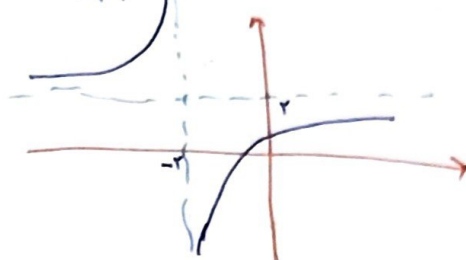
الف) $y = \sqrt{\frac{x+1}{x+1}} \rightsquigarrow R_f = \sqrt{\mathbb{R} - \{1\}} \rightarrow R_f = [0 + \infty) - \{\sqrt{1}\}$

ب) $y = \sqrt{\frac{x+1}{x-1}} \rightsquigarrow R_f = \sqrt{\mathbb{R} - \{-1\}} \rightarrow R_f = [0 + \infty)$

الف) $y = \frac{r_n - r}{n + 1}$



ب) $y = \frac{r_n - 1}{n + r}$



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الف) $y = \sin n + \frac{1}{\sin n} \rightsquigarrow R_f = (-\infty, -r] \cup [r, +\infty)$

ب) $y = \frac{n^4 + 1}{n^2} \rightsquigarrow y = n^2 + \frac{1}{n^2} \rightarrow R_f = (-\infty, -r] \cup [r, +\infty)$

ج) $y = \frac{\sqrt{n^2 + 1}}{\sqrt{n}} \rightsquigarrow y = \sqrt{n} + \frac{1}{\sqrt{n}} \rightarrow R_f = (-\infty, -r] \cup [r, +\infty)$

د) $y = \sqrt{n} + \frac{1}{\sqrt{n}} \rightsquigarrow y = \sqrt{n} + \frac{1}{\sqrt{n}} \rightsquigarrow R_f = [r, +\infty)$

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الف) $y = n^r + \frac{1}{n^r + r}$

$y = n^r + r + \frac{1}{n^r + r} - r$

R_f

$\rightarrow R_f = [\frac{1}{r} + \infty) - r \rightarrow R_f = [\frac{1}{r} + \infty)$

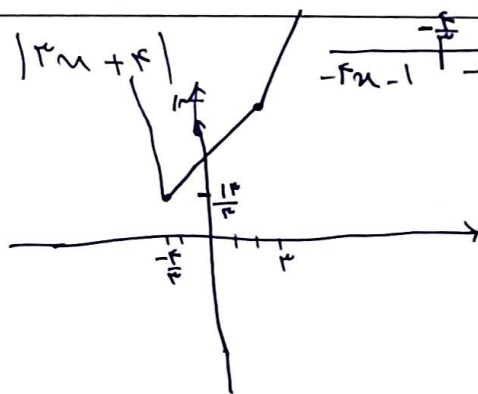
ب) $y = \frac{n^r + r}{\sqrt{n^r + r}}$

$y = \frac{n^r + r}{\sqrt{n^r + r}} + \frac{1}{\sqrt{n^r + r}} = \sqrt{n^r + r} + \frac{1}{\sqrt{n^r + r}}$

$\rightsquigarrow R_f = [\frac{2}{r} + \infty)$

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$y = |n - r| + |rn + r|$

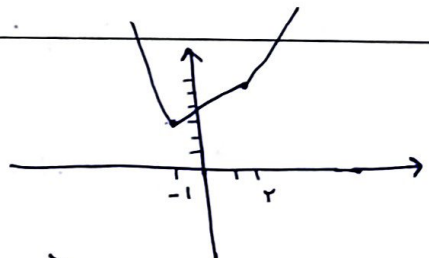


$\frac{-rn - 1}{-n + r + rn + r} \quad \frac{r}{n - r + rn + r}$

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الف) $y = |n - r| + |rn + r|$

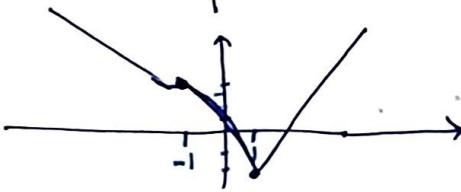
$R_f = [r, +\infty)$



ب) $y = |rn - r| - |n + 1|$

$\frac{-1}{-n + r} \quad \frac{1}{rn + 1} \quad x - r$

$R_f = [-r, +\infty)$



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