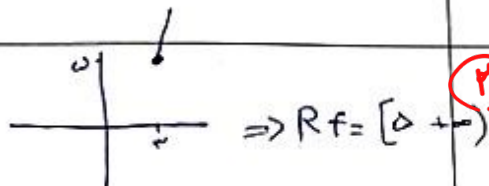


$$f(-1) = 0 \quad f(1) = 0 \\ f(0) = 1 \quad f(2) = 0$$

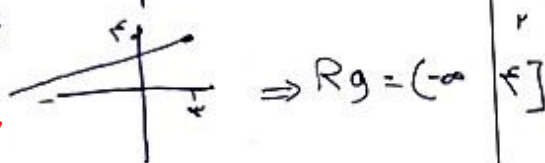
$$\Rightarrow Rg - Rf = \{(-1, 2) (0, 5) (1, 4)\} \Rightarrow Rg - Rf = \{2, 4, 5\}$$

$$\Rightarrow 2 + 4 + 5 = 11 \checkmark$$

$$f(x) = 2x - 1 \quad x \geq 2 \Rightarrow f(2) = 3$$

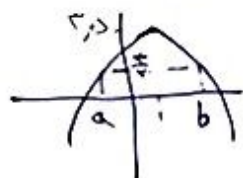


$$g(x) = \frac{1}{3}x + 2 \quad x \leq 2 \Rightarrow g(2) = \frac{5}{3}$$



$$\Rightarrow Rf \cup Rg = (-\infty, 5/3] \cup [3, +\infty) \checkmark$$

$$x_{max} = \frac{-b}{2a} = \frac{-1}{-1} = 1 \Rightarrow y_{max} = 2$$

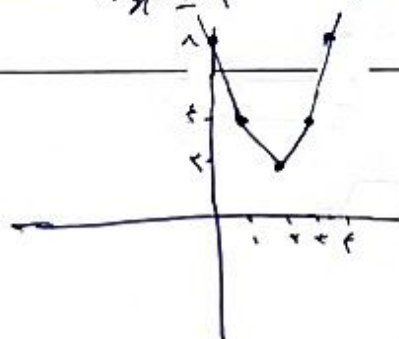


$$-\frac{x^2}{2} + x + 2 > \frac{3}{2} \Rightarrow -\frac{x^2}{2} + x + \frac{1}{2} > 0$$

$$a + c = b \Rightarrow \begin{cases} x = -1 \\ x = 3 \end{cases} \quad \frac{-1 \pm 2}{-1 \pm 1} \Rightarrow \begin{cases} a = -1 \\ b = 3 \end{cases}$$

$$\Rightarrow \sqrt{b-a} = \sqrt{4} = 2 \checkmark$$

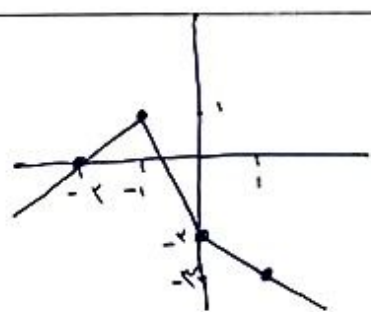
$$\begin{aligned} x=1 &\Rightarrow y=2 \\ x=3 &\Rightarrow y=2 \\ x=2 &\Rightarrow y=1 \\ x=0 &\Rightarrow y=2 \\ x=4 &\Rightarrow y=1 \end{aligned}$$



$$\Rightarrow Rf = [1, +\infty)$$

$$\Rightarrow \boxed{\min = 1} \checkmark$$

$$\begin{aligned} x=0 &\Rightarrow y=-2 \\ x=-1 &\Rightarrow y=1 \\ x=-2 &\Rightarrow y=0 \\ x=1 &\Rightarrow y=-3 \end{aligned}$$



$$\Rightarrow Rf = (-\infty, 1] \checkmark$$

$$y^2 + y = x^2 - 5x + m \Rightarrow x^2 - 5x + m - y^2 - y = 0 \Rightarrow x^2 + (-5-y)x + m-y = 0$$

$$\Rightarrow x = \frac{-(-5-y) \pm \sqrt{(-5-y)^2 - 4(m-y)}}{2} \Rightarrow y^2 + 7y + 25 - 4m \geq 0$$

آنچه $m = 4$ باشد از بزرگترین ضریب

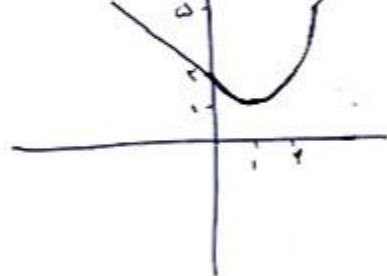
و دیگر ۱۸ نیست پس $m = 1, 2, 3, 4$

به ازای مقدار طبیعی

$$14m \leq 45 \Rightarrow m \leq 4$$

$$m = 1, 2, 3, 4$$

$$x^2 - 2x + 2 \rightarrow x_{\min} = 1 \Rightarrow y_{\min} = 1 \quad x \in [1, +\infty)$$



$$\Rightarrow R_f = [1, +\infty) \checkmark$$

$$x^2 + 4x + 3 \rightarrow x_{\min} = -2 \Rightarrow y_{\min} = -1 \quad x \in [-2, -1]$$



$$\Rightarrow R_f = [-1, +\infty) \checkmark$$

$$2x + 3 \geq 0 \Rightarrow x \geq -\frac{3}{2}$$

$$Df = [b, +\infty) \rightarrow b = -\frac{3}{2} \checkmark$$

$$\text{if } x = b, a+1 = a \Rightarrow a = 4 \checkmark$$

$$\Rightarrow ab = 4 \times -\frac{3}{2} = -6 \checkmark$$

$$f(1) = 2\sqrt{2} \rightarrow f(1) + g(1) = 2\sqrt{2} \Rightarrow g(1) = \sqrt{2}$$

$$f(-1) = 4\sqrt{2} \rightarrow f(-1) + g(-1) = 2\sqrt{2} \Rightarrow g(-1) = -\sqrt{2}$$

$$\frac{f(x)}{9} - \frac{g(x)}{4} \xrightarrow{\min} \frac{f(1)}{9} - \frac{g(1)}{4} = \frac{2\sqrt{2}}{9} - \frac{\sqrt{2}}{4} = 0$$

$$\xrightarrow{\max} \frac{f(-1)}{9} - \frac{g(-1)}{4} = \frac{4\sqrt{2}}{9} - (-\frac{\sqrt{2}}{4}) = \sqrt{2}$$

$$\Rightarrow \left(\frac{f(x)}{9} - \frac{g(x)}{4} \right) \in [0, \sqrt{2}] \checkmark$$