ازدین مکالیان یازدهم A

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$f(x) = \sqrt{1-x^2} \rightarrow 1 - x^2 \geq 0 \Rightarrow x^2 \leq 1 \Rightarrow -1 \leq x \leq 1 \Rightarrow D_f = [-1, 1]$

$g = \{(-1,1), (0,4), (2,0), (1,2)\} \Rightarrow D_g = \{-1, 0, 2, 1\}$

$D_f \cap D_g \Rightarrow \{-1, 0, 1\}$

$f(1) = 0 \;,\; g(1) = 2 \Rightarrow (2g - 3f)(1) = 4$

$f(0) = 1 \;,\; g(0) = 4 \Rightarrow (2g - 3f)(0) = 5$

$f(-1) = 0 \;,\; g(-1) = 1 \Rightarrow (2g - 3f)(-1) = 2$

$\Rightarrow 2g - 3f = \{(1,4), (0,5), (-1,2)\}$

$\Rightarrow$ مجموع اعضای برد $= 4 + 5 + 2 = 11$

۲-

$f(x) = 2x - 1 \;,\; D_f = [3, +\infty) \xrightarrow[\text{شیب } +]{\text{خطی}} R_f = [5, +\infty)$ ، $(2\times 3) - 1$

$g(x) = \frac{1}{2}x + 2 \;,\; D_g = (-\infty, 3) \xrightarrow[\text{شیب } +]{\text{خطی}} R_g = (-\infty, 4]$

$\xrightarrow{R_f \cup R_g} (-\infty, 4] \cup [5, +\infty)$

۳-

$y = -\frac{x^2}{2} + x + 3$

$-\frac{x^2}{2} + x + 3 > \frac{3}{2} \Rightarrow -\frac{x^2}{2} + x + \frac{3}{2} > 0$

$\Rightarrow (x+1)(-x+3) > 0 \Rightarrow$

	-1		3	
$-$	ϕ	$+$	ϕ	$-$

$(-1, 3)$

(a, b)

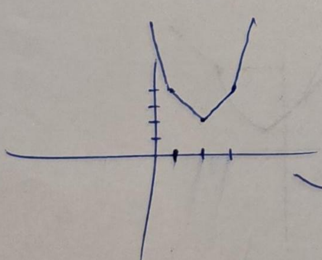
$a = -1 \;,\; b = 3 \Rightarrow \sqrt{b-a} = \sqrt{4} = 2$

۴-

$y = |x-1| + |x-3| + |2x-4|$

1 3 2

	1	2	3	
$-x+1-x+3$ $-2x+4$ $= -4x+8$		$x-1-x+3-2x$ $+4$ $= -2x+6$	$x-1-x+3$ $+2x-4$ $= 2x-2$	$x-1+x-3+2x-4$ $= 4x-8$
	4	2	4	



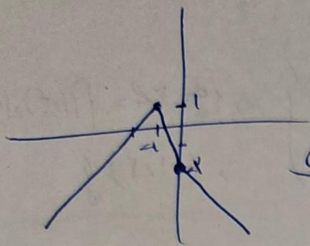
$\longrightarrow \min = 2$

۵-

$y = |x| - 2|x+1|$

$0 \qquad -1$

-1		0
$-x+2x+2 = x+2$	$-x-2x-2 = -3x-2$	$x-2x-2 = -x-2$
1		-2



$\xrightarrow{\text{طبق شکل}} R_y = (-\infty, 1]$

۶-

$y = \dfrac{x^2+5x+m}{x+1} \Rightarrow xy + y = x^2+5x+m \Rightarrow x^2+(5-y)x+m-y=0$

$\Rightarrow x = \dfrac{y-5 \pm \sqrt{y^2-10y+25+4y-4m}}{2} \rightarrow y^2-6y+25-4m \geq 0$

$\Rightarrow 36-100+16m \leq 0 \Rightarrow 16m-64 \leq 0 \Rightarrow \left.\begin{matrix} m \leq 4 \\ m \in N \end{matrix}\right\} m = \underbrace{\{1,2,3,4\}}$

۴ مقدار طبیعی برای m

۸-

$f(x) = \begin{cases} x^2+4x+3 \ ; & x \leq 0 \\ [2x]-2x \ ; & x>0 \end{cases} \overset{2x=a}{\Rightarrow} y_2 = [a]-a \Rightarrow R_{y_2} = (-1, 0]$

$y_1 = x^2+4x+3 \Rightarrow x_s = \dfrac{-b}{2a} = -2 \qquad \Rightarrow y_1 = x^2+4x+3 \quad \left\{ a>0 \right.$

$y_s = 4-8+3 = -1 \qquad ext \left| \begin{matrix} -2 \\ -1 \end{matrix} \right. \qquad \Rightarrow R_{g_1} = [-1, +\infty)$

$R_{y_1} \cup R_{y_2} \rightarrow R_f = [-1, +\infty)$

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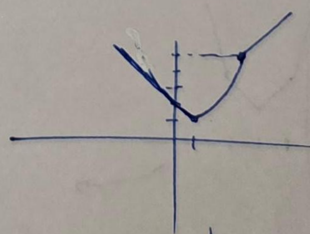
۷-

$f(x) = \begin{cases} x+2 \ ; & x \geq 3 \overset{x=3}{\Rightarrow} y=5 \\ x^2-2x+2 \ ; & 0 \leq x < 3 \overset{x=0}{\Rightarrow} y=2 \\ & \overset{x=3}{\Rightarrow} y=5 \\ |x|+2 & x<0 \overset{x=0}{\Rightarrow} y=2 \end{cases}$

$y = x^2-2x+2 \Rightarrow x_s = -\dfrac{b}{2a} = -\dfrac{-2}{2} = 1$

$y_s = 1$



$R_f = [1, +\infty)$

9-

$y = a+1-\sqrt{2x+3} \Rightarrow 2x+3 \geq 0 \Rightarrow x \geq -\frac{3}{2} \Rightarrow D_y = [-\frac{3}{2}, +\infty)$

$D_g = [b, +\infty)$ (I) $\Rightarrow b = -\frac{3}{2}$ $\quad ab = -6$

$R_g = (-\infty, d)$ (II) $\quad a = 4$

$\Rightarrow -\infty < a+1-\sqrt{2x+3} \leq a+1 \Rightarrow R_g = (-\infty, \underset{5}{\underbrace{a+1}})$

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$f(x) = 2\sqrt{x+1} + 4\sqrt{1-x} \overset{\mathrm{I} \cap \mathrm{II}}{\Rightarrow} D_f = [-1, 1] = D_g = D_y$

$f(x) + g(x) = 3\sqrt{2+2\sqrt{1-x^2}} \Rightarrow g(x) = 3\sqrt{2+2\sqrt{1-x^2}} - f(x) = 3\sqrt{2+2\sqrt{1-x^2}} - 2\sqrt{x+1}$
$+ 4\sqrt{1-x}$

$$y = \frac{f(x)}{6} - \frac{g(x)}{3} = \frac{2\sqrt{x+1} + 4\sqrt{1-x}}{6} - \frac{3\sqrt{2+2\sqrt{1-x^2}} - 2\sqrt{x+1} - 4\sqrt{1-x}}{3}$$

$$= \frac{\sqrt{x+1}}{3} + \frac{2\sqrt{x+1}}{3} + \frac{2\sqrt{1-x}}{3} + \frac{4\sqrt{x-1}}{3} - \sqrt{2+2\sqrt{1-x^2}} = \sqrt{x+1} + 2\sqrt{1-x} - \sqrt{(\sqrt{1-x}+\sqrt{1+x})^2}$$

$$= \sqrt{x+1} + 2\sqrt{1-x} - |\sqrt{1-x} + \sqrt{1+x}| = \sqrt{x+1} + 2\sqrt{1-x} - \sqrt{1-x} - \sqrt{1+x} = \sqrt{1-x}$$

$D_g = [-1, 1] \Rightarrow R_g = [0, \sqrt{2}]$