

$$f(x) = \sqrt{1-x^2} \Rightarrow 1-x^2 \geq 0$$

$$-1 \leq x \leq 1$$

(1 د)

$$g = \{(-1,1), (0,1), (1,1)\}$$

$$\Rightarrow (fg - f^2)(-1) = f(-1) = \sqrt{1-(-1)^2} = 1$$

$$fg - f^2 = \{(-1,1), (0,1), (1,1)\}$$

$$\Rightarrow (fg - f^2)(0) = f(0) = \sqrt{1-(0)^2} = 1$$

$$\Rightarrow (fg - f^2)(1) = f(1) = \sqrt{1-(1)^2} = 0$$

$$f(x) \cdot \lim_{x \rightarrow 1} \Rightarrow \lim_{x \rightarrow 1} f(x) = f(1) = 0$$

(2 د)

$$g(x) = \frac{1}{x}x + 1 \Rightarrow \lim_{x \rightarrow 1} g(x) = \frac{1}{1}(1) + 1 = 2 \Rightarrow \lim_{x \rightarrow 1} g(x) = 2$$

$$\lim_{x \rightarrow 1} f(x) = 0$$

$$y = -\frac{x^2}{2} + x + \frac{1}{2} \Rightarrow (a, b) = \left(\frac{1}{2}, \frac{5}{8}\right)$$

(3 د)

$$-\frac{x^2}{2} + x + \frac{1}{2} > \frac{1}{2} \Rightarrow -\frac{x^2}{2} + x > 0 \Rightarrow x(-\frac{x}{2} + 1) > 0 \Rightarrow x < 0 \text{ or } x > 2$$

$$\begin{array}{c|cc} x & -1 & 1 \\ \hline f(x) & + & - \end{array}$$

$$(-1, 2) \Rightarrow \sqrt{b-a} = \sqrt{2-(-1)} = \sqrt{3} = 2$$

$$y = |x-1| + |x-2| + |x-3|$$

(4 د)

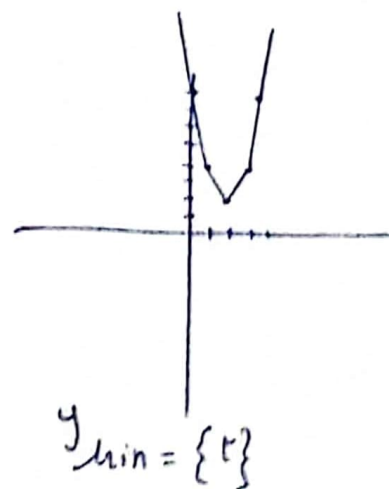
$$\Rightarrow y = |(-1)-1| + |(-1)-2| + |(-1)-3| = 0 + 1 + 1 = 2$$

$$\Rightarrow y = |(1)-1| + |(1)-2| + |(1)-3| = 0 + 1 + 1 = 2$$

$$\Rightarrow y = |(2)-1| + |(2)-2| + |(2)-3| = 1 + 0 + 1 = 2$$

$$\Rightarrow y = |(3)-1| + |(3)-2| + |(3)-3| = 2 + 1 + 0 = 3$$

$$\Rightarrow y = |(0)-1| + |(0)-2| + |(0)-3| = 1 + 2 + 3 = 6$$



$$y = |x| - 2|x+1|$$

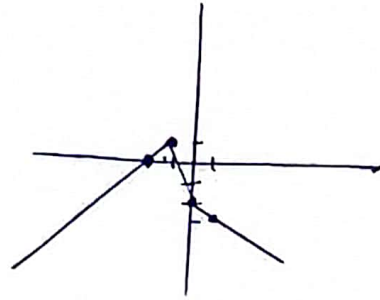
سوال ۵

$$x=0 \Rightarrow y = |0| - 2|0+1| = 0 - 2 = -2$$

$$x=-1 \Rightarrow y = |-1| - 2|(-1)+1| = 1 - 2(0) = 1$$

$$x=1 \Rightarrow y = |1| - 2|1+1| = 1 - 2(2) = -3$$

$$x=-2 \Rightarrow y = |-2| - 2|(-2)+1| = 2 - 2(1) = 0$$



$$y = \frac{x^2 + 5x + 4}{x+1} \quad \mathcal{D}_f = \mathbb{R} \Rightarrow \text{در صورتی که } x = -1 \text{ باشد این عبارت تعریف نشده خواهد شد}$$

برای این صحت دارد (طبیعی) بر تابع برای $\mathbb{R}$ نخواهد بود

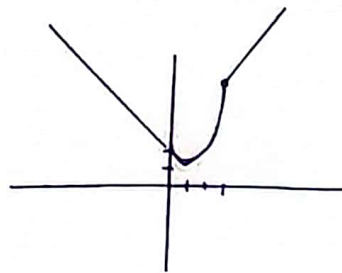
سوال ۶

$$f(x) = \begin{cases} x+2 & ; x > 2 \\ x^2-2x+2 & ; 0 \leq x \leq 2 \\ |x|+2 & ; x < 0 \end{cases}$$

$$x_{ext} : -\frac{b}{2a} = -\frac{5}{2(1)} = -\frac{5}{2}$$

$$y_{ext} : (1)^2 - 2(1) + 2 = 1$$

$$x=2 \Rightarrow (2)^2 - 2(2) + 2 = 2$$



$$\Rightarrow \mathcal{D}_f = [1, +\infty)$$

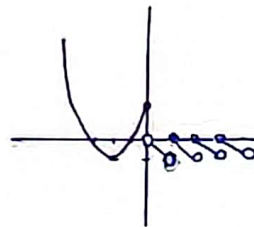
سوال ۷

$$f(x) = \begin{cases} x^2 + 4x + 4 & ; x \leq 0 \\ [x] - 2x & ; x > 0 \end{cases}$$

$$x_{ext} : -\frac{b}{2a} = -\frac{4}{2(1)} = -2$$

$$y_{ext} : (-2)^2 + 4(-2) + 4 = -1$$

$$x^2 + 4x + 4 = (x+2)^2 = 0 \Rightarrow x = -2$$



$$[x] - 2x \Rightarrow x = a$$

$$[a] - a \in (-1, 0]$$

$$\mathcal{D}_f = [-1, +\infty)$$

سوال ۸

$$y = a+1 - \sqrt{2x+3}$$

$$\mathcal{D}_f = [b, +\infty)$$

$$\mathcal{D}_f = (-\infty, 5] \Rightarrow \mathcal{D}_f$$

$$a+1 = 5 \Rightarrow a = 4$$

$$ab = (4)\left(-\frac{3}{2}\right) = -6$$

سوال ۹

$$x > -\frac{3}{2}$$

این تابع دارای بیشترین مقدار
این تابع زمانی است که زیر را دنبال کنیم

$$f(x) = \mu\sqrt{x+1} + \mu\sqrt{1-x} \quad D_f = D_g = -1 \leq x \leq 1 (I)$$

(سوال 10)

$$f(x) + g(x) = \mu\sqrt{\mu + \mu\sqrt{1-x}} = \mu\sqrt{(\sqrt{1-x} + \sqrt{1+x})^2} = \mu(\sqrt{1-x} + \sqrt{1+x})$$

$$\mu\sqrt{x+1} + \mu\sqrt{1-x} + g(x) = \mu\sqrt{1-x} + \mu\sqrt{1+x} \Rightarrow g(x) = \sqrt{x+1} - \sqrt{1-x}$$

$$y = \frac{f(x)}{x} - \frac{g(x)}{\mu} = \frac{\mu\sqrt{x+1} + \mu\sqrt{1-x}}{x} - \frac{\sqrt{x+1} - \sqrt{1-x}}{\mu} = \frac{\mu\sqrt{x+1} + \mu\sqrt{1-x} - \sqrt{x+1} + \sqrt{1-x}}{\mu}$$

$$x=1 \Rightarrow \sqrt{1-(1)} = 0$$

$$= \frac{\mu\sqrt{1-x}}{\mu} = \sqrt{1-x}$$

$$\sqrt{1-x} \quad I$$

$$\Rightarrow x=-1$$

$$\sqrt{1-(-1)} = \sqrt{2}$$

$$\Rightarrow R_f = [0, \sqrt{2}]$$

