

17

موضوع: علی عمران - کلف 9

$$f\left(\frac{r}{4}\right) = \cos \frac{r}{4} \sqrt{3}$$

$$f(\sqrt{r}) = \sqrt{r+1} \quad \sqrt{r+1} \leq 2 \quad \checkmark \quad (2)$$

$$g\left(\frac{r}{4}\right) = 2 \cos^2 \frac{r}{4} = \frac{1}{2}$$

$$f\left(\frac{1}{r}\right) = \sqrt{\frac{r-1}{r}} = \frac{\sqrt{3}}{2} \quad \checkmark$$

(2)

$$g(\sqrt{r}) = \frac{\sqrt{r}}{1-\sqrt{r}} \times \frac{1+\sqrt{r}}{1+\sqrt{r}} = \frac{\sqrt{r+1}}{-1} = -(\sqrt{r+1})$$

$$f(-(\sqrt{r+1})) = \left[-\left(\frac{\sqrt{r+1}}{2}\right) \right] = -\frac{r}{4} \quad \checkmark$$

$$f\left(\frac{r}{4}\right) = \sin \frac{r}{4} = \frac{\sqrt{r}}{2}$$

$$g\left(\frac{\sqrt{r}}{2}\right) = \frac{\sqrt{r}}{2} \sqrt{1-\frac{1}{r}} = \frac{\sqrt{r}}{2} \times \frac{1}{\sqrt{r}} = \frac{1}{2} \quad \checkmark$$

(2)

$$1) f(g(x)) = \{(1,1)(1,1)(1,1)(1,1)\} \quad - 1$$

$$2) g(f(x)) = \{\} = \emptyset \quad \checkmark$$

$$3) f(f(x)) = \{\} = \emptyset \quad \checkmark$$

(2)

$$4) g(g(x)) = \{(1,1)(1,1)(1,1)\}$$

$$f(g(x)) \rightarrow f(g(1)) = 1 \rightarrow g(1) = 1 \quad - 1$$

$\rightarrow a = 1$ (2)

$$g(f(x)) \rightarrow g(f(1)) = 1 \quad b = 1$$

$$g(f(x)) = 1 \quad f(x) = 1 \rightarrow g(1) = 1 \rightarrow f(1) = 1 \rightarrow b = 1$$

$$f(g(x)) = 1 \quad f(x) = 1 \rightarrow g(1) = 1 \rightarrow a = 1$$

$$g(x+y) = x+y \rightarrow x+y \rightarrow x+y \rightarrow x+y \rightarrow x+y$$

$$f(x) = x \rightarrow f(x) = x \rightarrow f(x) = x \rightarrow f(x) = x \rightarrow f(x) = x$$

$$x = ax + b \rightarrow f(f(x)) = a(ax+b) + b = a^2x + ab + b \quad (2)$$

$$x \rightarrow a \rightarrow a \rightarrow ab + b \rightarrow ab + b \rightarrow ab + b \rightarrow ab + b$$

$$ab + b \rightarrow ab + b \rightarrow ab + b$$

$$f(f(1)) \leftarrow f(1) = 1 \rightarrow g(1) = 1 \quad \checkmark$$

$$f(1) = 1 \rightarrow g(1) = 1$$

$$D_{g \circ f} = \{x \in D_f \mid f(x) \in D_g\}$$

$$D_f = \mathbb{R}$$

$$D_g = \mathbb{R} - \{0, 1\}$$

$$g(f(x)) = \frac{1}{(x+|x|)^2 - 1} \sqrt{x+|x|}$$

$$f(x) \neq 0, 1$$

$$|x| \begin{cases} x > 0 \\ x = 0 \\ x < 0 \end{cases}$$

عبر كل قبول

$$\begin{cases} \sqrt{x+|x|} \neq 0 \rightarrow x > 0 \\ \sqrt{x+|x|} \neq 1 \rightarrow x \neq 1 \end{cases}$$

$$\forall x > 0 \rightarrow g(f(x)) = \frac{1}{x^2 - 1} \sqrt{2x} \rightarrow \neq 0$$

$$x^2 \neq \sqrt{2x} \rightarrow x^4 \neq 2x \rightarrow x^3 \neq 2 \rightarrow x \neq \sqrt[3]{2}$$

$$\rightarrow D_f \subset [0, +\infty) - \{\sqrt[3]{2}\} \quad D_{g \circ f} = (0, +\infty) - \{1\}$$

$$f+g = \sqrt{1-x^2} + \sqrt{x}$$

$$(f+g)(f(x)) = \sqrt{1-(\sqrt{1-x^2})^2} + \sqrt{\sqrt{1-x^2}} = \sqrt{x^2} + \sqrt[4]{1-x^2}$$

$$= |x| + \sqrt[4]{1-x^2} \rightarrow 1-x^2 \geq 0 \rightarrow x \in [-1, 1] \quad (y)$$

$$\rightarrow D_f \subset [-1, 1] \quad \checkmark$$

$$\text{ب) } \frac{x+1}{x-2} = f \rightarrow x = \frac{f+1}{f-2} \rightarrow f(f) = f\left(\frac{f+1}{f-2}\right) + 1$$

$$\rightarrow f(x) = \frac{x^2-1}{x-2} \quad \checkmark$$

(y)

$$\Rightarrow f\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right) \left(x^2 + \frac{1}{x^2} - 1\right)$$

$$\Rightarrow \left(x + \frac{1}{x}\right) \left(\left(x + \frac{1}{x}\right)^2 - 3\right) = f\left(x + \frac{1}{x}\right)$$

$$\Rightarrow f(x) = x(x^2 - 3) = x^3 - 3x \quad \checkmark$$

$$f(x) = x\sqrt{x}$$

$$g \circ f(1) = 0 \Rightarrow f(x) = 1 \Rightarrow x = 1$$

$$g \circ f(\sqrt{x}) = 0 \Rightarrow f(x) = \sqrt{x} \Rightarrow x = 1$$

$$\left. \begin{array}{l} f(x) = 1 \\ f(x) = \sqrt{x} \end{array} \right\} \{1, 1\} \quad \textcircled{y}$$