

$$f(x) = \begin{cases} \cot \frac{\pi x}{4} & ; x \leq 1 \\ \sqrt{x^2 + 1} & ; x > 1 \end{cases}$$

$$f \circ f\left(\frac{\sqrt{3}}{2}\right) \Rightarrow \cot \frac{\frac{\sqrt{3}}{2} \pi}{4} = \cot \frac{\pi \sqrt{3}}{8} = \sqrt{3}$$

$$f(\sqrt{3}) = \sqrt{3^2 + 1} = 2$$

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الف) $f\left(\frac{1}{x}\right) = \sqrt{\frac{2x-1}{x^2}}$, $g(x) = \cos^2 x$ $f \circ g\left(\frac{\pi}{4}\right) \Rightarrow g\left(\frac{\pi}{4}\right) = \cos^2 \frac{\pi}{4} = \frac{1}{2}$

$$f\left(\frac{1}{2}\right) \Rightarrow f\left(\frac{1}{2}\right) = \sqrt{\frac{2 \cdot \frac{1}{2} - 1}{\left(\frac{1}{2}\right)^2}} \rightarrow t = \frac{1}{2} \rightarrow x = \frac{1}{t} \Rightarrow f(t) = \sqrt{2t - t^2} \rightarrow f\left(\frac{1}{2}\right) = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$$

ب) $f(x) = [x]$, $g(x) = \frac{x}{1-x} \Rightarrow f \circ g(\sqrt{2}) \Rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{-1} = -\sqrt{2}-2$

$$f(-\sqrt{2}-2) = [-\sqrt{2}-2] = -4$$

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$f(x) = \sin x$, $g(x) = x\sqrt{1-x^2}$, $g \circ f\left(\frac{\pi}{4}\right) = g\left(f\left(\frac{\pi}{4}\right)\right)$

$$\Rightarrow f\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \rightarrow g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \times \sqrt{1 - \left(\frac{\sqrt{2}}{2}\right)^2}$$

$$g \circ f\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \times \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{2}{4} = \frac{1}{2}$$

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$f(x) = \{(t, \omega), (y, \omega), (\lambda, 12), (10, 12)\}$, $g(x) = \{(t, y), (y, \omega), (\omega, \lambda), (\lambda, 10)\}$

الف) $f \circ g(x) = \{(t, \omega), (y, \omega), (\omega, 12), (\lambda, 12)\}$

ب) $g \circ f(x) = \{(t, y), (y, \omega), (\omega, \lambda), (\lambda, 10)\}$

ج) $f \circ f(x) = \{(t, \omega), (y, \omega), (\omega, 12), (\lambda, 12)\}$

د) $g \circ g(x) = \{(t, \lambda), (y, y), (\omega, \omega), (\lambda, 10)\}$

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$f = \{(1, 1), (3, 2), (4, \omega), (1, 7)\}$, $g = \{(1, 2), (3, 1), (a, 3), (b, 1)\}$

$(1, 2) \in f \circ g$, $(1, 1) \in g \circ f$

$\{(1, 1) \in g \circ f \rightarrow g(f(1)) = 1 \xrightarrow{f(1)=\omega} g(\omega) = 1 \rightarrow b = \omega$
 $\{(1, 2) \in f \circ g \rightarrow f(g(1)) = 2 \xrightarrow{f(2)=1} g(1) = 2 \rightarrow a = 1$

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$$f\left(\frac{1}{x}\right) = \frac{1}{x+1} \quad g\left(\frac{1}{x+1}\right) = \frac{1}{x-1} \quad g \circ f(-1)$$

$$\frac{1}{x+1} = t \Rightarrow x = \frac{t-1}{1} \quad g(t) = \frac{1}{\frac{t-1}{1}-1} = \frac{1}{t-2} \Rightarrow g(x) = \frac{1}{x-2}$$

$$f(x) = ax+b \quad f(ax+b) = a(ax+b)+b = a^2x+ab+b = a^2x+(ab+b) = \frac{1}{x}+1$$

$$a^2 = 1 \rightarrow a = 1 \vee a = -1$$

$$ab+b = 1 \begin{cases} a=1 \rightarrow b=1 \rightarrow f(x) = \frac{1}{x}+1 \\ a=-1 \rightarrow b=-1 \rightarrow f(x) = -\frac{1}{x}-1 \end{cases} \rightarrow f(-1) = -1$$

$$g(f(-1)) = g(-1) = \frac{1}{-1-2} = -\frac{1}{3}$$

$$f(x) = \sqrt{x+|x|}, \quad g(x) = \frac{1}{x^2-1}$$

$$D_f = \begin{cases} \sqrt{x} & ; x \geq 0 \\ 0 & ; x < 0 \end{cases} \rightarrow D_f = \mathbb{R}$$

$$D_g \Rightarrow x(x-1) \neq 0 \Rightarrow x \neq 0, 1$$

$$f(x) \neq 0, f \begin{cases} f(x) \neq 0 \rightarrow (0, +\infty) \\ f(x) \neq 1 \rightarrow \mathbb{R} - \{1\} \end{cases}$$

$$\Rightarrow D_{(g \circ f)} = (0, +\infty) - \{1\}$$

$$f(x) = \sqrt{1-x^2}, \quad g(x) = \sqrt{x}$$

$$D_{(f+g) \circ f} = \{x \in D_f : f(x) \in D_{(f+g)}\}$$

$$D_f \Rightarrow 1-x^2 \geq 0 \Rightarrow x \in [-1, 1]$$

$$D_g \Rightarrow x \geq 0 \Rightarrow D_g = [0, +\infty)$$

$$D_{(f+g)} = [0, 1]$$

$$D_{(f+g) \circ f} = [-1, 1]$$

$$f\left(\frac{x+1}{x-1}\right) = \frac{1}{x+1}$$

$$\frac{x+1}{x-1} = t \Rightarrow t(x-1) = x+1 \Rightarrow x = \frac{t+1}{t-1}$$

$$f(t) = f\left(\frac{t+1}{t-1}\right) = \frac{1}{\frac{t+1}{t-1}+1} = \frac{t-1}{t+2}$$

$$f(x) = \frac{1}{x+1}$$

$$f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x}$$

$$x + \frac{1}{x} = t \Rightarrow \left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 \Rightarrow \left(x^2 + \frac{1}{x^2}\right) = t^2 - 2$$

$$f(t) = t^2 - 2$$

$$f(x) = x^2 - 2$$

$$f(x) = 1 \Rightarrow x\sqrt{x} = 1 \Rightarrow x = 1$$

$$f(x) = 1\sqrt{1} \Rightarrow x\sqrt{x} = 1\sqrt{1} \Rightarrow x = 1$$

$$\Rightarrow 1-1 = 0$$