

$$f(x) = \begin{cases} \cot \frac{\pi x}{4} & ; x \leq 1 \\ \sqrt{x^2 + 1} & ; x > 1 \end{cases}$$

$$f \circ f\left(\frac{\sqrt{3}}{2}\right) \Rightarrow \cot \frac{\frac{\sqrt{3}}{2} \pi}{4} = \cot \frac{\pi \sqrt{3}}{8} = \sqrt{3}$$

$$f(\sqrt{3}) = \sqrt{3^2 + 1} = 2$$

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الف) $f\left(\frac{1}{x}\right) = \sqrt{\frac{1-x}{x^2}}$, $g(x) = \cos^2 x$ $f \circ g\left(\frac{\pi}{4}\right) \Rightarrow g\left(\frac{\pi}{4}\right) = \cos^2 \frac{\pi}{4} = \frac{1}{2}$
 $f\left(\frac{1}{2}\right) \Rightarrow f\left(\frac{1}{x}\right) = \sqrt{\frac{1-x}{x^2}} \rightarrow t = \frac{1}{x} \rightarrow x = \frac{1}{t} \Rightarrow f(t) = \sqrt{1-t^2} \rightarrow f\left(\frac{1}{2}\right) = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$

ب) $f(x) = [x]$, $g(x) = \frac{x}{1-x} \Rightarrow f \circ g(\sqrt{2}) \Rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{-1} = -\sqrt{2}-2$
 $f(-\sqrt{2}-2) = [-\sqrt{2}-2] = -4$

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$$f(x) = \sin x , g(x) = x\sqrt{1-x^2} , g \circ f\left(\frac{\pi}{4}\right) = g\left(f\left(\frac{\pi}{4}\right)\right)$$

$$\Rightarrow f\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \rightarrow g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \sqrt{1-\frac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$g \circ f\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \times \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$f(x) = \{(t, \omega), (y, \omega), (\lambda, \mu), (\iota, \mu)\} , g(x) = \{(t, \gamma), (\nu, \epsilon), (z, \lambda), (\lambda, \iota)\}$$

الف) $f \circ g(x) = \{(t, \omega), (y, \omega), (z, \mu), (\lambda, \mu)\}$ $\begin{cases} t \rightarrow \gamma \rightarrow \omega \\ \nu \rightarrow \epsilon \rightarrow \omega \\ y \rightarrow \lambda \rightarrow \mu \\ \lambda \rightarrow \iota \rightarrow \mu \end{cases}$

ب) $g \circ f(x) = \{(t, \gamma), (y, \epsilon), (\lambda, \mu), (\iota, \mu)\}$ $\begin{cases} t \rightarrow \omega \rightarrow x \\ y \rightarrow \omega \rightarrow x \\ \lambda \rightarrow \mu \rightarrow x \\ \iota \rightarrow \mu \rightarrow x \end{cases}$

ج) $f \circ f(x) = \{(t, \omega), (y, \omega), (\lambda, \mu), (\iota, \mu)\}$ $\begin{cases} t \rightarrow \omega \rightarrow x \\ y \rightarrow \omega \rightarrow x \\ \lambda \rightarrow \mu \rightarrow x \\ \iota \rightarrow \mu \rightarrow x \end{cases}$

د) $g \circ g(x) = \{(t, \gamma), (\nu, \epsilon), (z, \lambda), (\lambda, \iota)\}$ $\begin{cases} t \rightarrow \gamma \rightarrow \lambda \\ \nu \rightarrow \epsilon \rightarrow \gamma \\ y \rightarrow \lambda \rightarrow \iota \end{cases} \quad \lambda \rightarrow \iota \rightarrow x$

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$$f = \{(1, 1), (3, 2), (4, \omega), (1, \nu)\} \quad g = \{(1, 2), (3, 1), (a, 3), (b, 1)\}$$

$$(1, 2) \in f \circ g , (1, 1) \in g \circ f$$

$$\begin{cases} (1, 1) \in g \circ f \rightarrow g(f(1)) = 1 \xrightarrow{f(1)=\omega} g(\omega) = 1 \rightarrow b = \omega \\ (1, 2) \in f \circ g \rightarrow f(g(1)) = 2 \xrightarrow{f(2)=1} g(1) = 2 \rightarrow a = 1 \end{cases} \Rightarrow (a, b) = (1, \omega)$$

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$$f\left(\frac{1}{x}\right) = \frac{1}{x+1} \quad g\left(\frac{1}{x+1}\right) = \frac{1}{x+1} - 1 \quad g \circ f(-1)$$

$$\frac{1}{x+1} = t \Rightarrow x = \frac{t-1}{1} \quad g(t) = \frac{1}{t} - 1 = \frac{1}{t-1} - 1 \Rightarrow g(x) = \frac{1}{x} - 1$$

$$f(x) = ax+b \quad f(ax+b) = a(ax+b) + b = a^2x + ab + b = a^2x + (ab+b) = f^2(x) + 1$$

$$a^2 = f \rightarrow a = \sqrt{f} \quad a = -1$$

$$ab+b = 1 \quad \begin{cases} a=1 \rightarrow b=1 \rightarrow f(x) = x+1 \rightarrow f(-1) = -1 \\ a=-1 \rightarrow b=-1 \rightarrow f(x) = -x-1 \end{cases} \quad g(f(-1)) = g(-1) = \frac{1}{-1} - 1 = -2$$

$$f(x) = \sqrt{x+|x|} \quad g(x) = \frac{1}{x^2 - x} \quad D_f = \begin{cases} \sqrt{x} & ; x \geq 0 \\ 0 & ; x < 0 \end{cases} \rightarrow D_f = \mathbb{R}$$

$$D_g \Rightarrow x(x-1) \neq 0 \Rightarrow x \neq 0, 1$$

$$f(x) \neq 0, f \begin{cases} f(x) \neq 0 \rightarrow (0, +\infty) \\ f(x) \neq f \rightarrow \mathbb{R} - \{1\} \end{cases}$$

$$\Rightarrow D(g \circ f) = (0, +\infty) - \{1\}$$

$$D(g \circ f) = \{x \in D_f : f(x) \in D_g\}$$

$$f(x) = \sqrt{1-x^2} \quad g(x) = \sqrt{x} \quad D_{(f+g) \circ f} = \{x \in D_f : f(x) \in D_{(f+g)}\}$$

$$D_f \Rightarrow 1-x^2 \geq 0 \Rightarrow x \in [-1, 1] \rightarrow D_f = [-1, 1] \quad D_{f+g} = [0, 1] \quad x \in [-1, 1] \rightarrow 0 \leq \sqrt{1-x^2} \leq 1 \checkmark$$

$$D_g \Rightarrow x \geq 0 \rightarrow D_g = [0, +\infty)$$

$$D_{(f+g) \circ f} = [-1, 1]$$

$$f\left(\frac{1}{x+1}\right) = \frac{1}{x+1} \quad \frac{1}{x+1} = t \Rightarrow t(x+1) = 1 \Rightarrow x = \frac{1-t}{t}$$

$$f(t) = f\left(\frac{1}{t}\right) + 1 = \frac{1}{t} + 1 = \frac{1+t}{t} \rightarrow f(t) = \frac{1+t}{t} \Rightarrow f(x) = \frac{1+x}{x}$$

$$f\left(x + \frac{1}{x}\right) = x + \frac{1}{x} \quad x + \frac{1}{x} = t \Rightarrow \left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 = t^2 \Rightarrow \left(x + \frac{1}{x}\right)^2 = t^2 - 2$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = t^2 - 2 \quad x + \frac{1}{x} = t \rightarrow f(t) = t^2 - 2 \rightarrow f(x) = x^2 - 2$$

$$f(x) = 1 \rightarrow x\sqrt{x} = 1 \Rightarrow x = 1$$

$$f(x) = 1\sqrt{1} \rightarrow x\sqrt{x} = 1\sqrt{1} \Rightarrow x = 1$$

$$\Rightarrow 1-1=0 \quad \text{انتقال تابع}$$

زبان که دارای تابع g و h است
با یک مقدار تابع f در h است
با یک مقدار h و یک مقدار g است