

۱۷/۵

$$f(f(\frac{1}{2})) = f(6 + \frac{1}{2}\pi) = f(\sqrt{3}) \Rightarrow f(\sqrt{3}) = \sqrt{(\sqrt{3})^2 + 1} = \sqrt{4} = 2$$

(۲)

الف) $f \circ g(\frac{\pi}{4}) \Rightarrow g(\frac{\pi}{4}) = 2 \sin^2(\frac{\pi}{4}) = 2(\frac{1}{2})^2 = \frac{1}{2}$ (۱)
 $\Rightarrow f(\frac{1}{2}) = \sqrt{\frac{2(2)-1}{2}} = \sqrt{\frac{3}{2}}$ ✓

ب) $g \circ f(\sqrt{2}) \Rightarrow f(\sqrt{2}) = [\sqrt{2}] = 1$ $f \circ g(\sqrt{2}) = ? \rightarrow g(\sqrt{2}) = -\sqrt{2} - 2$
 $\Rightarrow g(1) = \frac{1}{1-1} = \frac{1}{0}$ تعریف نشده $f(-\sqrt{2}-2) = -4$

$$g \circ f(\frac{\pi}{2}) = g(f(\frac{\pi}{2})) \Rightarrow f(\frac{\pi}{2}) = \sin \frac{\pi}{2} = \frac{\sqrt{2}}{2}$$

$$\Rightarrow g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2} (\sqrt{1 - (\frac{\sqrt{2}}{2})^2}) = \frac{\sqrt{2}}{2} \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2}$$

(۲)

الف) $f \circ g(n) = f(g(n)) = \{(4,5), (2,5), (4,12), (8,12)\}$ ✓

ب) $g \circ f(x) = g(f(x)) = \emptyset$ ✓

ج) $f \circ f(n) = \emptyset$ ✓

د) $g \circ g(n) = \{(4,8), (2,4), (4,10)\}$ ✓

(۲)

$f \circ g \rightarrow (2,2) \rightarrow f(g(n)) = 2 \Rightarrow g(n) = 3 \Rightarrow \boxed{x = a = 3}$

$g \circ f \rightarrow (4,1) \rightarrow g(f(n)) = 1 \Rightarrow f(n) = 3 \times$

$f = \{(2,1), (3,2), (4,5), (1,7)\}$
 $g = \{(1,2), (2,1), (a,3), (b,1)\}$

$b \rightarrow f(4) = 5 \Rightarrow \boxed{b = 5}$

$\Rightarrow (a,b) \in (4,5)$ ✓

(۲)

$$f(f(x)) = a(ax+b)+b = a^2x + ab + b = 5x + 7 \Rightarrow \begin{cases} a = \pm 2 \\ b = 1 \text{ or } -7 \end{cases}$$

$$g(2x+7) = 2x-7 \rightarrow a'(2x+7)+b' = 2x-7 = 2a'x + a' + b' \Rightarrow \begin{cases} a' = 1/2 \\ b' = -4.5 \end{cases}$$

$$\Rightarrow g(f(-1)) \rightarrow \begin{cases} f(-1) = 2(-1) + 7 = 5 \\ f(-1) = -2(-1) - 7 = -5 \end{cases} \Rightarrow g(5) = \frac{1}{2}(5) - 4.5 = -2 \quad \text{or} \quad g(-5) = \frac{1}{2}(-5) - 4.5 = -5.5$$

$$f(x) = \sqrt{x+|x|} \rightarrow x+|x| \geq 0 \rightarrow x \geq 0 \rightarrow D_f = \mathbb{R}^+$$

$$g(x) = \frac{1}{x^2-2x} = \frac{1}{x(x-2)} \Rightarrow D_g = \mathbb{R} - \{0, 2\} \quad f(x) \neq 0, \quad \begin{cases} \sqrt{x+|x|} \neq 0 \rightarrow x > 0 \\ \sqrt{x+|x|} \neq 2 \rightarrow x \neq 4 \end{cases}$$

$$D_{f \circ g} = \{x \in D_g, g(x) \in D_f\} \rightarrow \mathbb{R} - \{0, 2, 4\} \rightarrow (0, +\infty) - \{4\}$$

$$f(x) = \sqrt{1-x^2} \rightarrow D_f = [-1, 1] \quad D_{f \circ g} = [0, 1] \checkmark$$

$$g(x) = \sqrt{x} \rightarrow D_g = [0, +\infty)$$

$$D_{f \circ g} = \{x \in D_g, g(x) \in D_f\} \Rightarrow \begin{cases} D_f = [-1, 1] \\ D_g = [0, +\infty) \end{cases} \rightarrow D_{f \circ g} = [0, 1] \checkmark$$

$$f(x) \in D_{f \circ g} \rightarrow -1 \leq \sqrt{1-x^2} \leq 1 \rightarrow x^2 \geq 0 \rightarrow D_f$$

$$\text{الف) } f\left(\frac{2x+1}{x-2}\right) = 5x+8 \xrightarrow{\frac{2x+1}{x-2} = t} 2x+1 = t(x-2) \Rightarrow x(t-2) = 2t-1 \Rightarrow x = \frac{2t-1}{t-2}$$

$$\rightarrow f(t) = 5x+8 \rightarrow f(t) = 5\left(\frac{2t-1}{t-2}\right) + 8 = \frac{13t-11}{t-2} \Rightarrow f(x) = \frac{13x-11}{x-2}$$

$$\text{ب) } f\left(x + \frac{1}{x^2}\right) = x^2 + \frac{1}{x^2} \Rightarrow x + \frac{1}{x} = t \rightarrow f(t) = x^2 + \frac{1}{x^2} \Rightarrow x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 \left(x + \frac{1}{x}\right) \Rightarrow f(t) = t^2 - 2t \Rightarrow f(x) = x^2 - 2x$$

$$g(x) = 1, \sqrt{2} \Rightarrow a(x-1)(x-\sqrt{2}) = ax^2 - (\sqrt{2}+1)ax + \sqrt{2}a$$

$$= ax^2 - a(\sqrt{2}+1)x + \sqrt{2}a \Rightarrow g(f(x)) = g(x\sqrt{x}) = a(x\sqrt{x})^2 - (\sqrt{2}+1)a(x\sqrt{x}) + \sqrt{2}a$$

$$= ax^2 - (\sqrt{2}+1)a(x\sqrt{x}) + \sqrt{2}a \xrightarrow{\text{مقارنة المعاملات}} \begin{cases} 1 \\ 2 \end{cases} \Rightarrow \sqrt{2}-1$$