

$$f(f(\frac{\pi}{2})) = f(\cos \frac{\pi}{2}) = f(0) = f(\sqrt{3}) \Rightarrow f(\sqrt{3}) = \sqrt{(\sqrt{3})^2 + 1} = \sqrt{4} = 2$$

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$$\text{الف) } f \circ g(\frac{\pi}{4}) \Rightarrow g(\frac{\pi}{4}) = 2 \sin^2(\frac{\pi}{4}) = 2(\frac{1}{2})^2 = \frac{1}{2}$$

$$\Rightarrow f(\frac{1}{2}) = \sqrt{2(\frac{1}{2}) - 1} = \sqrt{\frac{1}{2} - 1} = \sqrt{-\frac{1}{2}}$$

$$\text{ب) } g \circ f(\sqrt{2}) \Rightarrow f(\sqrt{2}) = [\sqrt{2}] = 1$$

$$\Rightarrow g(1) = \frac{1}{1-1} = \frac{1}{0} \rightarrow \text{تعریف نشده}$$

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$$g \circ f(\frac{\pi}{2}) = g(f(\frac{\pi}{2})) \Rightarrow f(\frac{\pi}{2}) = \sin \frac{\pi}{2} = 1$$

$$\Rightarrow g(1) = \frac{1}{1-1} = \frac{1}{0} \rightarrow \text{تعریف نشده}$$

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$$\text{الف) } f \circ g(n) = f(g(n)) = \{(1, 0), (2, 0), (4, 1), (8, 1)\}$$

$$\text{ب) } g \circ f(n) = g(f(n)) = \emptyset$$

$$\text{ج) } f \circ f(n) = \emptyset$$

$$\text{د) } g \circ g(n) = \{(1, 1), (2, 4), (4, 16)\}$$

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$$f \circ g \rightarrow (2, 1) \rightarrow f(g(n)) = 2 \Rightarrow g(n) = 3 \Rightarrow \boxed{x = a = 3}$$

$$g \circ f \rightarrow (4, 1) \rightarrow g(f(n)) = 1 \Rightarrow f(n) = 3 \times$$

$$f = \{(1, 1), (2, 2), (4, 4), (1, 1)\}$$

$$b \rightarrow f(4) = 8 \Rightarrow \boxed{b = 8}$$

$$g = \{(1, 2), (2, 1), (a, 3), (b, 1)\}$$

$$\Rightarrow (a, b) \in (4, 8)$$

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$$f(f(x)) = a(ax+b)+b = a^2x + ab + b = 5x + 7 \Rightarrow \begin{cases} a = \pm 2 \\ b = 1 \text{ or } -7 \end{cases}$$

$$g(2x+3) = 2x-1 \rightarrow a'(2x+3)+b' = 2x-1 = 2a'x + 3a' + b' \Rightarrow \begin{cases} a' = 1/2 \\ b' = -4/3 \end{cases}$$

$$\Rightarrow g(f(-1)) \rightarrow \begin{cases} f(-1) = 2x+1 = -1 \\ f(-1) = -2x-7 = -1 \end{cases} \Rightarrow g(-1) = \frac{1}{2}(-1) - 4/3 = -11/6$$

$$f(x) = \sqrt{x+|x|} \rightarrow x+|x| \geq 0 \rightarrow x \geq 0 \rightarrow D_f = \mathbb{R}$$

$$g(x) = \frac{1}{x^2-2x} = \frac{1}{x(x-2)} \Rightarrow D_g = \mathbb{R} - \{0, 2\}$$

$$D_{f \circ g} = \{x \in D_g, g(x) \in D_f\} \rightarrow \mathbb{R} - \{0, 2\}$$

$$f(x) = \sqrt{1-x^2} \rightarrow D_f = [-1, 1] \xrightarrow{\text{①}} D_{f \circ g} = [0, 1]$$

$$g(x) = \sqrt{x} \rightarrow D_g = [0, +\infty)$$

$$D_{f \circ g \circ f} = \{x \in D_f, D_f \in D_{g \circ f}\} \Rightarrow \begin{cases} D_f = [-1, 1] \\ D_{g \circ f} = [0, 1] \end{cases} \xrightarrow{\text{①}} D_{f \circ g \circ f} = [0, 1]$$

$$\text{الف) } f\left(\frac{2x+1}{x-2}\right) = 5x+8 \xrightarrow{\frac{2x+1}{x-2} = t} 2x+1 = t(x-2) \Rightarrow x(t-2) = 2t+1 \Rightarrow x = \frac{2t+1}{t-2}$$

$$\rightarrow f(t) = 5x+8 \rightarrow f(t) = 5\left(\frac{2t+1}{t-2}\right) + 8 = \frac{13t-11}{t-2} \Rightarrow f(x) = \frac{13x-11}{x-2}$$

$$\text{ب) } f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} \Rightarrow x + \frac{1}{x} = t \rightarrow f(t) = x^2 + \frac{1}{x^2} \Rightarrow x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 \left(x + \frac{1}{x}\right)$$

$$\Rightarrow f(t) = t^2 - 2t \Rightarrow f(x) = x^2 - 2x$$

$$g(x) \text{ is linear} = 1, 2\sqrt{2} \Rightarrow a(x-1)(x-2\sqrt{2}) = ax^2 - 2\sqrt{2}ax - ax + 2\sqrt{2}a$$

$$= ax^2 - a(2\sqrt{2}+1)x + 2\sqrt{2}a \Rightarrow g(f(x)) = g(x\sqrt{x}) = a(x\sqrt{x})^2 - (2\sqrt{2}+1)a(x\sqrt{x}) + 2\sqrt{2}a$$

$$= ax^2 - (2\sqrt{2}+1)a(x\sqrt{x}) + 2\sqrt{2}a \xrightarrow{\text{مقارنة معاملات}} \begin{cases} 1 \\ 2 \end{cases} \Rightarrow 2 - 1/2 = 1$$