

$$f(m) = \begin{cases} \cot \frac{\pi x}{4} & \text{if } x \leq 1 \\ \sqrt{x^2 + 1} & \text{if } x > 1 \end{cases}$$

$$\textcircled{I} f\left(\frac{1}{2}\right) = \cot \frac{\frac{1}{2}\pi}{4} = \cot \frac{\pi}{8} = \sqrt{2}$$

$$f \circ f\left(\frac{1}{2}\right) = \sqrt{2} \rightarrow f(f(m)) = f\left(f\left(\frac{1}{2}\right)\right) = \textcircled{II} f(\sqrt{2}) = \sqrt{(\sqrt{2})^2 + 1} = \sqrt{2+1} = \sqrt{3}$$

$$\text{الف)} \begin{cases} f\left(\frac{1}{x}\right) = \sqrt{\frac{x-1}{x^2}} \\ g(m) = 2 \cot \frac{\pi}{4} \end{cases}$$

$$f(g(m)) = \textcircled{I} g(m) \rightarrow g\left(\frac{1}{2}\right) = 2 \cot \frac{\pi}{4} = 2 \left(\frac{1}{2}\right) = 1$$

$$\text{ب)} \begin{cases} f(m) = [x] \\ g(m) = \frac{x}{1-x} \end{cases} \quad \text{fog}(\sqrt{2}) = ?$$

$$\textcircled{I} f(\sqrt{2}) = [\sqrt{2}] = 1$$

$$\textcircled{II} g(1) = \frac{1}{1-1} = \frac{1}{0} \rightarrow \text{undefined}$$

$$g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} = -\sqrt{2}-2$$

$$f(-\sqrt{2}-2) = -4$$

$$f(m) = \sin x$$

$$g(m) = x \sqrt{1-m^2}$$

$$g \circ f\left(\frac{\pi}{2}\right) = ? \rightarrow g(f(\frac{\pi}{2})) \rightarrow$$

$$\textcircled{I} f\left(\frac{\pi}{2}\right) = \sin \frac{\pi}{2} = 1 \rightarrow$$

$$\textcircled{II} g\left(\frac{1}{2}\right) = \frac{1}{2} \cdot \sqrt{1 - \left(\frac{1}{2}\right)^2} = \frac{1}{2} \cdot \sqrt{1 - \frac{1}{4}} = \frac{1}{2} \cdot \sqrt{\frac{3}{4}} = \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$$

$$f(m) = \{(f, \omega), (4, \omega), (1, 12), (10, 12)\} \quad \text{و} \quad g(m) = \{(f, 4), (2, f), (4, 1), (1, 10)\}$$

$$\text{الف)} f \circ g(m) \rightarrow f(g(m)) \rightarrow g(m) = \{4, f, 1, 10\} \Rightarrow f(4, f, 1, 10) = \{(f, \omega), (2, \omega), (4, 12), (1, 12)\}$$

$$\text{ب)} g \circ f(m) \rightarrow g(f(m)) \rightarrow f(m) = \{\omega, 12\} \Rightarrow g(\omega, 12) = \{\} = \emptyset$$

$$\text{ج)} f \circ f(m) \rightarrow f(f(m)) \rightarrow f(m) = \{\omega, 12\} \Rightarrow f(\omega, 12) = \{\} = \emptyset$$

$$\text{د)} g \circ g(x) \rightarrow g(g(x)) \Rightarrow g(m) = \{4, f, 1, 10\} \Rightarrow g(4, f, 1, 10) = \{(f, 1), (2, 4), (4, 10)\}$$

$$f = \{(2, 1), (3, 2), (f, \omega), (1, v)\}$$

$$g = \{(1, 2), (3, 1), (a, 3), (b, 1)\}$$

$$(f, 2) \in f \circ g \rightarrow f(g(m)) \rightarrow \{2, 1, 3\}$$

$$(f, 1) \in g \circ f \rightarrow g(f(m)) \rightarrow 1, 2, \omega, v \rightarrow (b, 1) \Rightarrow b = \omega$$

$$(a, b) = ?$$

خواب و بیداری و ...

$$f(f(m)) = f(m+1) \rightarrow a(am+b)+b = a^2m+ab+b = f(m+1)$$

$$g(2m+1) = 3m-1 \rightarrow am+b \rightarrow \begin{cases} a=2 \\ b=1 \end{cases}$$

$$g \circ f(-1) = \dots \rightarrow cm+d \Rightarrow c(1m+1)+d = 3m-1 \rightarrow \begin{cases} c=1 \\ d=-2 \end{cases}$$

$$L \rightarrow g(f(-1)) \rightarrow \begin{cases} g(1) = 2(1)+1 = 3 \\ g(-1) = 2(-1)+1 = -1 \end{cases}$$

$$f(m) = \sqrt{x+|x|} \rightarrow D_f = \{x \mid x+|x| \geq 0\} \rightarrow D_f = \mathbb{R}$$

$$g(m) = \frac{1}{x^2 - 4x} \rightarrow D_g = \{x \mid x^2 - 4x \neq 0\} \rightarrow D_g = \mathbb{R} \setminus \{0, 4\}$$

$$D_{g \circ f} = \{x \mid x \in D_f \text{ and } f(x) \in D_g\} \rightarrow \{x \mid x \in \mathbb{R} \text{ and } \sqrt{x+|x|} \neq 0, 4\}$$

$$f(x) \neq 0, 4 \rightarrow \sqrt{x+|x|} \neq 0, 4 \rightarrow x+|x| \neq 0, 16 \rightarrow x \neq 0, -16$$

$$f(m) = \sqrt{1-x^2} \rightarrow D_f = \{x \mid 1-x^2 \geq 0\} \rightarrow D_f = [-1, 1]$$

$$g(m) = \sqrt{x} \rightarrow D_g = \{x \mid x \geq 0\}$$

$$D_{f+g} = D_f \cap D_g = [0, 1]$$

$$f\left(\frac{2x+1}{x-2}\right) = f(m) + \omega \rightarrow \frac{2x+1}{x-2} = t \rightarrow x = \frac{2t+1}{t-2} \rightarrow f(t) = f\left(\frac{2t+1}{t-2}\right) + \omega$$

$$f(m) = \frac{1}{m} \rightarrow f(m) = f\left(\frac{2x+1}{x-2}\right) + \omega \rightarrow f(m) = \frac{x-2}{2x+1} + \omega$$

$$f(x) = \frac{1}{x} \rightarrow f\left(x + \frac{1}{x}\right) = x + \frac{1}{x} \rightarrow \left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 = x^2 + \frac{1}{x^2} + 2 \left(x + \frac{1}{x}\right)$$

$$f(m) = m\sqrt{m}$$

$$g(m) = \frac{1}{2\sqrt{m}} \rightarrow (m-1)(m-2\sqrt{2}) = m^2 - 2\sqrt{2}m - m + 2\sqrt{2} = m^2 - (2\sqrt{2}+1)m + 2\sqrt{2}$$

$$g \circ f \rightarrow g(f(m)) = g(m\sqrt{m}) \rightarrow \frac{1}{2\sqrt{m\sqrt{m}}} = \frac{1}{2m^{3/4}}$$

$$x=1 \rightarrow \dots$$

$$x=2 \rightarrow \dots$$