

$$f(m) = \begin{cases} \cot \frac{\pi x}{4} & \text{if } x \leq 1 \\ \sqrt{x^2 + 1} & \text{if } x > 1 \end{cases}$$

$$\textcircled{I} f\left(\frac{1}{2}\right) = \cot \frac{\frac{1}{2}\pi}{4} = \cot \frac{\pi}{8} = \sqrt{2}$$



$$f \circ f\left(\frac{1}{2}\right) = \sqrt{2} \rightarrow f(f(m)) = f\left(f\left(\frac{1}{2}\right)\right) = \textcircled{II} f(\sqrt{2}) = \sqrt{(\sqrt{2})^2 + 1} = \sqrt{3}$$

$$\text{الف) } \begin{cases} f\left(\frac{1}{x}\right) = \sqrt{\frac{x-1}{x^2}} \\ g(m) = 2 \cot \frac{\pi}{4} \end{cases}$$

$$f(g(m)) = \textcircled{I} g(m) \rightarrow g\left(\frac{1}{2}\right) = 2 \cot \frac{\pi}{4} = 2 \left(\frac{1}{2}\right) = 1$$



$$\text{ب) } \begin{cases} f(m) = [x] \\ g(m) = \frac{x}{1-x} \end{cases}$$

$$\textcircled{I} f(\sqrt{2}) = [\sqrt{2}] = 1$$

$$g \circ f(\sqrt{2}) \rightarrow g(f(m)) = g(1) = \frac{1}{1-1} = \frac{1}{0} \rightarrow \text{undefined}$$

$$f(m) = \sin x$$

$$g(m) = x \sqrt{1-m^2}$$

$$g \circ f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} \rightarrow g(f(\frac{\pi}{2})) \rightarrow \textcircled{II} g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \cdot \sqrt{1 - \left(\frac{\sqrt{2}}{2}\right)^2} = \frac{\sqrt{2}}{2} \cdot \sqrt{1 - \frac{1}{2}} = \frac{\sqrt{2}}{2} \cdot \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2}$$



$$f(m) = \{(f, \omega), (4, \omega), (1, 12), (10, 12)\} \quad \text{و} \quad g(m) = \{(f, 4), (2, f), (4, 1), (1, 10)\}$$

$$\text{الف) } f \circ g(m) \rightarrow f(g(m)) \rightarrow g(m) = \{4, f, 1, 10\} \Rightarrow f(f, \omega, 4, 10) = \{(f, \omega), (2, \omega), (4, 12), (1, 12)\}$$

$$\text{ب) } g \circ f(m) \rightarrow g(f(m)) \rightarrow f(m) = \{\omega, 12\} \Rightarrow g(\omega, 12) = \{\} = \emptyset$$

$$\text{ج) } f \circ f(m) \rightarrow f(f(m)) \rightarrow f(m) = \{\omega, 12\} \Rightarrow f(\omega, 12) = \{\} = \emptyset$$

$$\text{د) } g \circ g(x) \rightarrow g(g(x)) \Rightarrow g(m) = \{4, f, 1, 10\} \Rightarrow g(4, f, 1, 10) = \{(f, 1), (2, 4), (4, 10)\}$$

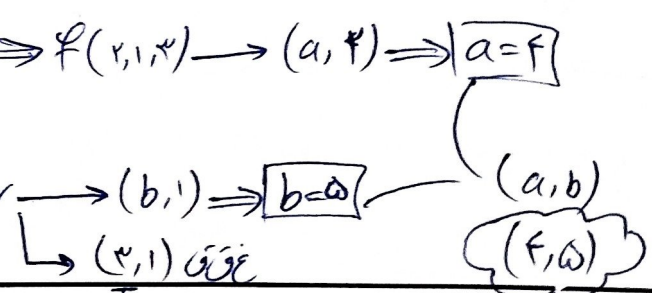
$$f = \{(2, 1), (3, 2), (f, \omega), (1, v)\}$$

$$g = \{(1, 2), (3, 1), (a, 3), (b, 1)\}$$

$$(f, 2) \in f \circ g \rightarrow f(g(m)) \rightarrow \{2, 1, 3\}$$

$$(f, 1) \in g \circ f \rightarrow g(f(m)) \rightarrow 1, 2, \omega, v \rightarrow (b, 1) \Rightarrow b = \omega$$

$$(a, b) = \{$$



خواب و بیداری و ...

$f(f(m)) = f(m+3) \rightarrow a(am+b)+b = a^2m+ab+b = f(m+3)$   
 $g(2m+3) = 3m-2 \rightarrow \begin{cases} am+b \\ cm+d \end{cases} \rightarrow \begin{cases} a < +3 \rightarrow b=1 \\ -r \rightarrow b=-3 \end{cases}$   
 $g \circ f(-1) = ? \rightarrow cm+d \Rightarrow c(2m+3)+d = 3m-2$   
 $\rightarrow g(f(-1)) \begin{cases} g < 2m+1 = 2(-1)+1 = -1 \rightarrow 2cm+3c+d = 3m-2 \rightarrow \begin{cases} c=+\frac{3}{2} \\ d=-\frac{13}{2} \end{cases} \\ -2m-3 = -2(-1)-3 = -1 \rightarrow g(-1) = \frac{3}{2}(-1) - \frac{13}{2} = -8 \end{cases}$

$f(m) = \sqrt{x+|x|} \rightarrow D_f = m+|m| \geq 0 \rightarrow \begin{cases} \oplus \\ \ominus \end{cases} \rightarrow D_f = \mathbb{R}$   
 $g(m) = \frac{1}{x^2 - 4x} \rightarrow D_g = x^2 - 4x \neq 0 \rightarrow x(x-4) \neq 0 \rightarrow x \neq 0, 4$   
 $D_{g \circ f} = ? \rightarrow D_{g \circ f} = \{x \in D_f \mid f(m) \in D_g\} \rightarrow \{ \mathbb{R} - \{0, 4\}, \mathbb{R} \}$   
 $\xrightarrow{\text{النتيجة}} \mathbb{R} - \{0, 4\} \cap \mathbb{R} \rightarrow \mathbb{R} - \{0, 4\}$

$f(m) = \sqrt{1-x^2} \rightarrow D_f \Rightarrow 1-x^2 \geq 0 \rightarrow x^2 \leq 1 \rightarrow -1 \leq x \leq 1$   
 $g(m) = \sqrt{x} \rightarrow D_g = [x \geq 0]$   
 $D_{(f+g) \circ f} = ?$   
 $\xrightarrow{\text{النتيجة}} D_{f+g} = D_f \cap D_g \Rightarrow \begin{matrix} \text{---} \bullet \text{---} \bullet \text{---} \\ -1 \quad 0 \quad +1 \end{matrix} \Rightarrow D_f \cap D_g = [0, 1] \text{ (I)}$   
 $\Rightarrow D_{(f+g) \circ f} = \{x \in D_f, f(m) \in D_{f+g}\} \rightarrow [0, 1] \cap [-1, 1] \rightarrow [0, 1]$

ا)  $f\left(\frac{2m+1}{x-3}\right) = f(m+5) \rightarrow \frac{2m+1}{x-3} = t \Rightarrow x = \frac{2t+1}{t-3} \Rightarrow f(t) = f\left(\frac{2t+1}{t-3}\right) + 5$   
 $f(m) = ? \Rightarrow f(m) = f\left(\frac{2m+1}{x-3}\right) + 5 \rightarrow f(m) = \frac{2m+1}{x-3} + 5$   
 ب)  $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} \rightarrow \left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 = x^2 + \frac{1}{x^2} + 2 \left(x + \frac{1}{x}\right)$   
 $f(m) = ? \rightarrow x + \frac{1}{x} = t \rightarrow f(t) = t^2 - 2t \Rightarrow f(m) = m^2 - 2m$

$f(m) = m\sqrt{m}$   
 $g(m) = \begin{cases} 1 \\ 2\sqrt{2} \end{cases} \rightarrow \begin{cases} 1 \\ 2\sqrt{2} \end{cases} \rightarrow (m-1)(m-2\sqrt{2}) = m^2 - 2\sqrt{2}m - m + 2\sqrt{2} = m^2 - (2\sqrt{2}+1)m + 2\sqrt{2}$   
 $g \circ f \rightarrow g(f(m)) = g(m\sqrt{m}) \Rightarrow (m\sqrt{m})^2 - (2\sqrt{2}+1)(m\sqrt{m}) + 2\sqrt{2} = m^3 - (2\sqrt{2}+1)(m\sqrt{m}) + 2\sqrt{2}$   
 $x=1 \rightarrow \text{مجموع ضربات صفر است}$   
 $x=2 \rightarrow \text{اختلاف قبل تقاطع شده} = \text{اختلاف بعده} = 2-1 = 1$