

$$f\left(\frac{x}{3}\right) = \cot \frac{2\pi}{\frac{x}{3}} = \cot \frac{\pi}{\frac{x}{6}} = \sqrt{3}$$

$\frac{x}{3} < 1$

$$f \circ f\left(\frac{x}{3}\right) = f(\sqrt{3}) = \sqrt{3+1} = 2$$

$\sqrt{3} > 1$

الف) $g\left(\frac{\pi}{4}\right) = 2 \times \cos^2 \frac{\pi}{4} = \frac{1}{2} \rightarrow f\left(\frac{1}{2}\right) = \sqrt{\frac{4-1}{2}} = \sqrt{\frac{3}{2}} = \frac{\sqrt{3}}{2}$

ب) $g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} \cdot \frac{\sqrt{2}+2}{1-2} = \frac{\sqrt{2}+2}{-1} = -(\sqrt{2}+2) = -\sqrt{2}-2$

$$f(-\sqrt{2}-2) = [-\sqrt{2}-2] = [-\sqrt{2}] - 2 = -2 - 2 = -4$$

$$f\left(\frac{\pi}{6}\right) = \sin \frac{\pi}{6} = \frac{\sqrt{2}}{2}$$

$$g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \times \sqrt{1-\frac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$

الف) $f \circ g(x) = \{(4, 5), (2, 5), (4, 12), (8, 12)\}$

ب) $g \circ f(x) = \emptyset$

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د) $g \circ g(x) = \{(4, 8), (2, 4), (4, 10)\}$

$$(3, 2) \in f \circ g \Rightarrow \begin{matrix} 2 \in R_f \\ 3 \in D_g \end{matrix} \Rightarrow (a, 2) \rightarrow (3, 2) \Rightarrow a = 4$$

$$(4, 1) \in g \circ f \Rightarrow \begin{matrix} 1 \in R_g \\ 4 \in D_f \end{matrix} \Rightarrow (4, 2) \rightarrow (b, 1) \Rightarrow b = 5$$

$$\Rightarrow (a, b) = (4, 5)$$

$$f(x) = ax + b \Rightarrow f(f(x)) = a(ax + b) + b = f^2x + f$$

$$\Rightarrow ax^2 + ab + b = f^2x + f \Rightarrow \begin{cases} a^2 = f^2 \Rightarrow a = \pm f \\ b(a+1) = f \end{cases}$$

$$\text{if } a = f \Rightarrow b = 1 \Rightarrow f(x) = fx + 1$$

$$\text{if } a = -f \Rightarrow b = -f \Rightarrow f(x) = -fx - f$$

$$g(x) = cx + d \Rightarrow c(fx + f) + d = f^2x - f \Rightarrow fc + c + d = f^2 - f$$

$$\Rightarrow \begin{cases} fc = f^2 \Rightarrow c = f \\ fc + d = -f \Rightarrow \frac{f}{f} + d = -f \Rightarrow d = -\frac{f^2}{f} \Rightarrow g(x) = \frac{f}{f}x - \frac{f^2}{f} \end{cases} \quad \begin{matrix} f(-1) = -1 \\ \rightarrow g(-1) = \frac{-1}{-1} = 1 \end{matrix}$$

$$D_{g \circ f} = \{x \in D_f : f(x) \in D_g\}$$

$$D_f = \mathbb{R}$$

$$x^2 - fx = x(x-f) \Rightarrow x < f \Rightarrow D_g = \mathbb{R} - \{0, f\}$$

$$\Rightarrow D_{g \circ f} = \{x \in \mathbb{R}, \sqrt{x+|x|} \neq \{0, f\}\}$$

$$x > 0, x \neq f \Rightarrow D_{g \circ f} = (0, +\infty) - \{f\}$$

$$D_f = [-1, 1] \Rightarrow D_{f+g} = [0, 1]$$

$$D_g = [0, +\infty)$$

$$D_{(f+g) \circ f} = \{x \in D_f : f(x) \in D_{f+g}\} = \{-1 \leq x \leq 1, 0 \leq \sqrt{1-x^2} < 1\}$$

$$\Rightarrow 0 \leq 1 - x^2 < 1 \Rightarrow 1 \geq x^2 > 0 \Rightarrow -1 \leq x < 0 \cup 0 < x \leq 1$$

$$\Rightarrow D_{(f+g) \circ f} = [-1, 0) \cup (0, 1]$$

$$\omega) \frac{fx+1}{x-f} = t \Rightarrow fx+1 = t(x-f) \Rightarrow x(f-t) = -(ft+1)$$

$$\Rightarrow x = \frac{ft+1}{t-f} \Rightarrow ft = f\left(\frac{ft+1}{t-f}\right) + 0 = \frac{f^2t+1}{t-f} \Rightarrow f(x) = \frac{f^2x+1}{x-f}$$

$$\hookrightarrow f\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right)^f - f\left(x + \frac{1}{x}\right) \Rightarrow f(x) = x^f - fx$$

$$g \circ f = 0 \rightarrow \begin{matrix} f(x) = 1 \Rightarrow x\sqrt{x} = 1 \Rightarrow x = 1 \\ f(x) = \sqrt{x} \Rightarrow x\sqrt{x} = \sqrt{x} \Rightarrow x = 1 \end{matrix}$$

$$x-1 = 1$$