آرین ملکیان یازدهم A

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$$fof\left(\frac{2}{3}\right) = f\left(f\left(\frac{2}{3}\right)\right) = f(\sqrt{3}) = 2 \Rightarrow fof\left(\frac{2}{3}\right) = 2$$

$$f(x) = \begin{cases} \cot\frac{\pi x}{4} & ; x \leq 1 \\ \sqrt{x^2+1} & ; x > 1 \end{cases} \quad \xrightarrow{\frac{2}{3} \leq 1} f\left(\frac{2}{3}\right) = \cot\frac{\pi}{6} = \sqrt{3}$$

$$\xrightarrow[\sqrt{3} > 1]{} f(\sqrt{3}) = \sqrt{4} = 2$$

۲، الف)

$$fog\left(\frac{\pi}{3}\right) = f\left(g\left(\frac{\pi}{3}\right)\right) = f\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2} \qquad fog\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$g(x) = 2\cos^2 x \xrightarrow{x=\frac{\pi}{3}} 2\cos^2\frac{\pi}{3} = 2\left(\frac{1}{2}\right)^2 = \frac{1}{2}$$

$$f\left(\frac{1}{x}\right) = \sqrt{\frac{2x-1}{x^2}} \xrightarrow[\frac{1}{x} = \frac{1}{2} \Rightarrow x = 2]{} f\left(\frac{1}{2}\right) = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

ب)

$$fog(\sqrt{2}) = f(g(\sqrt{2})) = f(-2-\sqrt{2}) = -4$$

$$g(x) = \frac{x}{1-x} \xrightarrow{x=\sqrt{2}} g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} = \frac{\sqrt{2}(1+\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})} = -(2+\sqrt{2}) = -2-\sqrt{2}$$

$$f(x) = [x] \xrightarrow{x=-2-\sqrt{2}} f(-2-\sqrt{2}) = [-2-\sqrt{2}] \underset{\sqrt{2} \approx 1.4}{=} [3{,}4] = -4$$

$$fog(\sqrt{2}) = -4$$

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$$gof\left(\frac{\pi}{4}\right) = g\left(f\left(\frac{\pi}{4}\right)\right) = g\left(\frac{\sqrt{2}}{2}\right) = \frac{1}{2}$$

$$f(x) = \sin x \xrightarrow{x=\frac{\pi}{4}} f\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$g(x) = x\sqrt{1-x^2} \xrightarrow[x=\frac{\sqrt{2}}{2}]{} g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}\sqrt{1-\frac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$gof\left(\frac{\pi}{4}\right) = \frac{1}{2}$$

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الف)

$$fog(x) = \begin{cases} 4 \to 6 \to 5 \\ 2 \to 4 \to 5 \\ 6 \to 8 \to 12 \\ 8 \to 10 \to 12 \end{cases}$$

ب) $gof(x) = \emptyset$

$4 \to 5 \to \times$

$6 \to 5 \to \times$

$8 \to 12 \to \times$

$10 \to 12 \to \times$

ج) $fof(x) = \emptyset$

$4 \to 5 \to \times$

$6 \to 5 \to \times$

$8 \to 12 \to \times$

$10 \to 12 \to \times$

۱) $gog(x) = \{(۴,۸),(۲,۶),(۶,۱۰)\}$

$۴ \to ۶ \to ۸$

$۲ \to ۴ \to ۶$

$۶ \to ۸ \to ۱۰$

$۸ \to ۱۰ \to \times$

۱۵

$(۴,۲) \in fog$

$۱ \to ۲ \to ۱$

$۳ \to ۴ \to ۷$

$a \to ۳ \to ۲$

$b \to ۱ \to ۷$

$fog(x) = \{(۱,۱),(۳,۷),(a,۲),(b,۷)\}$

$(۴,۱) \in gof$

$۲ \to ۱ \to ۲$

$۳ \to ۲ \to ۳$

$۴ \to b \to ۱$

$۱ \to ۷$

$\Rightarrow (۴,۲) = (a,۲) \Rightarrow a = ۴$

$\Rightarrow (۴,b) = ۴,۵ \Rightarrow b = ۵$

$f = \{(۲,۱),(۳,۲),(۴,۵),(۱,۷)\}$

$\Rightarrow (a,b) = (۴,۵)$

۱۶

$f(x) = ax+b \to$ خطی

$f(f(x)) = f(ax+b) = a^۲x + ab + b = ۴x+۳ \Rightarrow a^۲ = ۴ \Rightarrow a = \pm ۲$

$ab+b = ۳ \xRightarrow{a=۲} b = ۱$

$\xRightarrow{a=-۲} b = -۳$

$f(x) = ax+b \xRightarrow{a=۲,b=۱} f(x) = ۲x+۱ \xRightarrow{x=-۱} f(-۱) = -۱$

$\xRightarrow{a=-۲,b=-۳} f(x) = -۲x-۳ \xRightarrow[x=-۱]{} f(-۱) = -۱$ $\Bigg\} \Rightarrow f(-۱) = -۱$

$gof(-۱) = g(f(-۱)) = g(-۱) = -۸$

$g(۲x+۳) = ۳x-۲ \xRightarrow[x=-۲]{} g(-۱) = -۸$

$gof(-۱) = -۸$

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$D_{gof} = \{x \mid \underbrace{x \in D_f}_{x\in R} ; f(x) \in D_g\}$

$f(x) = \sqrt{x+|x|} \Rightarrow x+|x| \geq 0 \Rightarrow |x| \geq -x$ ✓

$\Rightarrow D_f = R$

$g(x) = \dfrac{1}{x^۲-۴x} \Rightarrow x^۲-۴x \neq 0 \Rightarrow x(x-۴) \neq 0$

$\Rightarrow x \neq 0,۴ \Rightarrow D_g = R - \{0,۴\}$

$\begin{cases} \sqrt{x+|x|} \neq 0 \\ \Rightarrow x+|x| \neq 0 \Rightarrow |x| \neq -x \\ \Rightarrow x > 0 \\ \sqrt{x+|x|} \neq ۴ \Rightarrow x+|x| \neq ۱۶ \end{cases}$

$0 \neq ۱۶$ ✓ | $۲x \neq ۱۶$ $\boxed{x \neq ۸}$

$\Rightarrow D_{gof} = (0,+\infty) - \{۸\}$

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$D_{(f+g)\circ f} = \{x \mid \underbrace{x \in D_f}_{x \in [-1,1]} ; \underbrace{f(x) \in D_{g+f}}\}$

$\downarrow$

$\sqrt{1-x^2} \in [0,1]$

$\Rightarrow \sqrt{1-x^2} \leq 1 \Rightarrow x^2 \geq 0 \Rightarrow x \in \mathbb{R}$

$D_{f+g} = D_f \cap D_g \Rightarrow D_{f+g} = [0,1]$

$g(x) = \sqrt{x} \Rightarrow x \geq 0 \Rightarrow D_g\,[0,+\infty)$

$f(x) = \sqrt{1-x^2} \Rightarrow (1-x)(1+x) \geq 0$ $\begin{array}{ccccc} & -1 & & 1 & \\ - & & + & & - \end{array}$ $\Rightarrow D_f = [-1,1]$

$\Rightarrow D_{(f+g)\circ f} = [0,1]$

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الف) $f\left(\frac{3x+1}{x-2}\right) = 4x+5$

$\frac{3x+1}{x-2} = t \Rightarrow x = \frac{2t+1}{t-3}$

$\Rightarrow f(t) = 4\left(\frac{2t+1}{t-3}\right) + 5 = \frac{8t+4}{t-3} + 5$

$= \frac{13t-11}{t-3} \Rightarrow f(x) = \frac{13x-11}{x-3}$

ب) $f\left(x+\frac{1}{x}\right) = x^3 + \frac{1}{x^3} \xRightarrow{x+\frac{1}{x}=t} f(t) = t^3 - 3t \Rightarrow f(x) = x^3 - 3x$

$x^3 + \frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3\left(x\left(\frac{1}{x}\right)\right)\left(x+\frac{1}{x}\right) \Rightarrow x^3 + \frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right)$

$\xRightarrow{x+\frac{1}{x}=t} x^3 + \frac{1}{x^3} = t^3 - 3t$

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$g(f(x)) = 0$

$g(1) = 0$

$g(2\sqrt{2}) = 0$

$\Rightarrow f(x) = 1 = x\sqrt{x} \xRightarrow{\text{توان ۲}} x^3 = 1 \Rightarrow x = 1$

یا $f(x) = 2\sqrt{2} = x\sqrt{x} \xRightarrow{\text{توان ۲}} x^3 = 8 \Rightarrow x = 2$

$\Rightarrow 2 - 1 = 1$