

$$\frac{y}{x} < 1 \Rightarrow f\left(\frac{y}{x}\right) = \cot \frac{\arctan \frac{y}{x}}{x} = \cot \frac{\arctan y}{x} = \sqrt{3}$$

$$\sqrt{3} > 1 \Rightarrow f \circ f\left(\frac{y}{x}\right) = f(\sqrt{3}) = \sqrt{(\sqrt{3})^2 + 1} = \sqrt{4} = 2$$

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الف) $f \circ g\left(\frac{x}{y}\right) = f\left(y \cos^2 \frac{x}{y}\right) = f\left(y \times \left(\frac{1}{y}\right)^2\right) = f\left(\frac{1}{y}\right) = \sqrt{\frac{y \times y - 1}{y^2}} = \sqrt{\frac{y^2 - 1}{y^2}} = \frac{\sqrt{y^2 - 1}}{y}$

ب) $f \circ g(\sqrt{2}) = f\left(\frac{\sqrt{2}}{1 - \sqrt{2}}\right) = f\left(\frac{\sqrt{2}}{1 - \sqrt{2}} \times \frac{1 + \sqrt{2}}{1 + \sqrt{2}}\right) = f\left(\frac{\sqrt{2}(1 + \sqrt{2})}{1 - 2}\right) = f(-2 - \sqrt{2}) = [-2 - \sqrt{2}] = -2 + [-\sqrt{2}]$

$\sqrt{2} = \sqrt{2} \Rightarrow -2 + [-\sqrt{2}] = -2 - \sqrt{2} = [-4]$

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$$g \circ f\left(\frac{x}{y}\right) = g\left(\sin \frac{x}{y}\right) = g\left(\frac{\sqrt{y}}{y}\right) = \frac{\sqrt{y}}{y} \sqrt{1 - \frac{1}{y}} = \frac{\sqrt{y}}{y} \sqrt{\frac{y-1}{y}} = \frac{\sqrt{y}}{y} \times \frac{1}{\sqrt{y}} = \frac{1}{y}$$

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- الف) $\{(4, 5), (4, 12), (1, 12)\}$ ✓
- ب) $\{\}$ ✓
- ج) $\{\}$ ✓
- د) $\{(4, 1), (4, 4), (4, 10)\}$ ✓

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$(f, y) \in f \circ g \Rightarrow g(f) = f^{-1}(y) \Rightarrow g(f) = 3 \Rightarrow (f, 3) \in g \Rightarrow a = f$

$(f, 1) \in g \circ f \Rightarrow f(f) = g^{-1}(1) \Rightarrow \omega = g^{-1}(1) \Rightarrow g(\omega) = 1 \Rightarrow (\omega, 1) \in g \Rightarrow b = \omega$

$(a, b) = (f, \omega)$

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$$y\alpha + \psi = t \Rightarrow g(t) = \psi\left(\frac{t-\psi}{y}\right) - \psi$$

$$f(x) = ax + b \Rightarrow f(f(x)) = a^2x + ab + b = f\alpha + \psi \Rightarrow a^2 = f, b(a+1) = \psi$$

$$\Rightarrow a = y, b = 1 \text{ ; } a = -y, b = -\psi$$

$$f(-1) = -a + b \Rightarrow -y + 1 = -1 \checkmark \Rightarrow g \circ f(-1) = g(-1) = \psi\left(\frac{-1-\psi}{y}\right) - \psi = \boxed{-9}$$

$$g(-1) = -y - \psi = \boxed{-1}$$

$$x + |x| = \begin{cases} 0 & x \leq 0 \\ 2x & x \geq 0 \end{cases}$$

$$D_f = \mathbb{R} \checkmark$$

$$x^2 - \sqrt{x} \neq 0$$

$$x(x - \sqrt{x}) \neq 0 \Rightarrow x \neq 0, x \neq 1$$

$$\Rightarrow D_g = \mathbb{R} - \{0, 1\} \checkmark$$

$$\Rightarrow f(x) \neq 0, f$$

$$\Rightarrow x > 0, yx \neq y \Rightarrow x \neq 1$$

$$D_{g \circ f} = \mathbb{R}^+ - \{1\}$$

$$\hookrightarrow x > 0, x \neq 1$$

$$D_{f \circ g} = (0, +\infty) - \{1\}$$

$$1 - x^2 \geq 0 \Rightarrow x \in [-1, 1]$$

$$(f+g)(x) = \sqrt{1-x^2} + \sqrt{x}$$

$$D_{f+g} = [0, 1]$$

$$\Rightarrow f(x) \in [0, 1]$$

$$\Rightarrow 0 \leq \sqrt{1-x^2} \leq 1$$

$$\Rightarrow D_{(f+g) \circ f} = [-1, 1] \checkmark$$

$$y$$

$$\text{الف) } \frac{yx+1}{x-y} = t \Rightarrow x = -\left(\frac{yt-1}{t-y}\right) = \frac{yt+1}{t-y}$$

$$\Rightarrow f(t) = f\left(\frac{yt+1}{t-y}\right) + \omega = \frac{1t+f+\omega t-\omega}{t-y} = \frac{yt-11}{t-y}$$

$$f(x) = \frac{13x-11}{x-y} \checkmark$$

$$\therefore f\left(x + \frac{1}{x}\right) = x^y + \frac{1}{x^y} = \left(x + \frac{1}{x}\right)^y - \psi\left(x + \frac{1}{x}\right)$$

$$\Rightarrow f(x) = x^y - \psi x, x \in \mathbb{R} - (-y, y)$$

$$y$$

$$\left. \begin{aligned} g \circ f(x) &= g(x\sqrt{x}) = 0 \\ g(1) &= g(x\sqrt{x}) = 0 \end{aligned} \right\} \begin{aligned} x\sqrt{x} &= 1 \Rightarrow x=1 \\ x\sqrt{x} &= y \Rightarrow x=y \end{aligned}$$

$$|y-1| = \boxed{1} \checkmark$$

$$y$$