

$$\frac{y}{x} < 1 \Rightarrow f\left(\frac{y}{x}\right) = \cot \frac{\arctan \frac{y}{x}}{x} = \cot \frac{\arctan y}{x} = \sqrt{3}$$

$$\sqrt{3} > 1 \Rightarrow f \circ f\left(\frac{y}{x}\right) = f(\sqrt{3}) = \sqrt{(\sqrt{3})^2 + 1} = \sqrt{4} = 2$$

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$$\text{الف) } f \circ g\left(\frac{\pi}{4}\right) = f\left(4 \cos^2 \frac{\pi}{4}\right) = f\left(4 \times \left(\frac{1}{2}\right)^2\right) = f(1) = \sqrt{\frac{4 \times 1 - 1}{1}} = \sqrt{\frac{3}{1}} = \sqrt{3}$$

$$\text{ب) } f \circ g(\sqrt{2}) = f\left(\frac{\sqrt{2}}{1-\sqrt{2}}\right) = f\left(\frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}}\right) = f\left(\frac{\sqrt{2}(1+\sqrt{2})}{1-2}\right) = f(-2-\sqrt{2}) = [-2-\sqrt{2}] = -2+[-\sqrt{2}]$$

$$\stackrel{f=14}{=} -2+[-14] = -2-2 = -4$$

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$$g \circ f\left(\frac{\pi}{6}\right) = g\left(\sin \frac{\pi}{6}\right) = g\left(\frac{1}{2}\right) = \frac{\sqrt{2}}{2} \sqrt{1-\frac{1}{4}} = \frac{\sqrt{2}}{2} \sqrt{\frac{3}{4}} = \frac{\sqrt{2}}{2} \times \frac{1}{2} = \frac{1}{2}$$

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الف) $\{(4, 5), (4, 12), (1, 12)\}$

ب) $\{\}$

ج) $\{\}$

د) $\{(4, 8), (4, 4), (4, 10)\}$

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$$(f, 4) \in f \circ g \Rightarrow g(f) = f^{-1}(4) \Rightarrow g(f) = 3 \Rightarrow (f, 3) \in g \Rightarrow a = f$$

$$(f, 1) \in g \circ f \Rightarrow f(f) = g^{-1}(1) \Rightarrow \omega = g^{-1}(1) \Rightarrow g(\omega) = 1 \Rightarrow (\omega, 1) \in g \Rightarrow b = \omega$$

$$(a, b) = (f, \omega)$$

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$$y\alpha + \psi = t \Rightarrow g(t) = \psi\left(\frac{t-\psi}{y}\right) - \psi$$

$$f(x) = ax + b \Rightarrow f(f(x)) = a^2x + ab + b = f\alpha + \psi \Rightarrow a^2 = f, b(a+1) = \psi$$

$$\Rightarrow a = y, b = 1 \text{ ; } a = -y, b = -\psi$$

$$f(-1) = -a + b \Rightarrow \begin{cases} -y + 1 = -1 \\ y - \psi = -1 \end{cases} \Rightarrow g \circ f(-1) = g(-1) = \psi\left(\frac{-1-\psi}{y}\right) - \psi = \boxed{-9}$$

$$x + |x| = \begin{cases} 0 & x \leq 0 \\ 2x & x \geq 0 \end{cases} \quad \begin{matrix} x - x = 0 \\ x + x = 2x \end{matrix}$$

$$D_f = \mathbb{R}$$

$$x^2 - \sqrt{x} \neq 0 \quad x(x - \sqrt{x}) \neq 0 \Rightarrow x \neq 0 \Rightarrow D_g = \mathbb{R} - \{0\}$$

$$\Rightarrow f(x) \neq 0, f \Rightarrow x > 0, \forall x \neq \sqrt{x} \Rightarrow D_{g \circ f} = \boxed{\mathbb{R}^+ - \{1\}}$$

$$1 - x^2 \geq 0 \Rightarrow x \in [-1, 1]$$

$$(f+g)(x) = \sqrt{1-x^2} + \sqrt{x}$$

$$D_{f+g} = [0, 1]$$

$$\Rightarrow f(x) \in [0, 1]$$

$$\Rightarrow 0 \leq \sqrt{1-x^2} \leq 1 \quad \text{مربعات جذر را}$$

$$\Rightarrow D_{(f+g) \circ f} = \boxed{[-1, 1]}$$

$$\text{الف) } \frac{\psi x + 1}{x - \psi} = t \Rightarrow x = -\left(\frac{-\psi t - 1}{t - \psi}\right) = \frac{\psi t + 1}{t - \psi}$$

$$\Rightarrow f(t) = f\left(\frac{\psi t + 1}{t - \psi}\right) + \omega = \frac{\Lambda t + f + \omega t - \omega}{t - \psi} = \frac{13t - 11}{t - \psi}$$

$$f(x) = \frac{13x - 11}{x - \psi}$$

$$\therefore f\left(x + \frac{1}{x}\right) = x^\psi + \frac{1}{x^\psi} = \left(x + \frac{1}{x}\right)^\psi - \psi\left(x + \frac{1}{x}\right)$$

$$\Rightarrow \boxed{f(x) = x^\psi - \psi x}, \quad x \in \mathbb{R} - (-\psi, \psi)$$

$$\left. \begin{aligned} g \circ f(x) &= g(x\sqrt{x}) = 0 \\ g(1) &= g(x\sqrt{x}) = 0 \end{aligned} \right\} \begin{aligned} x\sqrt{x} &= 1 \Rightarrow \boxed{x=1} \\ x\sqrt{x} &= \psi\sqrt{x} \Rightarrow \boxed{x=\psi} \end{aligned}$$

$$|\psi - 1| = \boxed{1}$$