

$$\cos \frac{\pi x}{4} \xrightarrow{(\frac{\pi}{4})} \cos \frac{\pi}{4} \rightarrow \cos 45^\circ \rightarrow \frac{\cos 45^\circ}{\sin 45^\circ} \rightarrow \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \sqrt{3} \quad \underline{12} - 1$$

$$f(\sqrt{3}) \rightarrow \sqrt{(\sqrt{3})^2 + 1} \rightarrow (2) \quad f \circ f\left(\frac{1}{\sqrt{3}}\right) = 2 \quad \text{بدلیل نوشتن اسم غرض شفافیت لحاظ می گردد} \quad \textcircled{2}$$

$$g(\frac{\pi}{4}) = 2 \cos^2 \frac{\pi}{4} \rightarrow 2 \times \frac{1}{2} = 1$$

$$f\left(\frac{1}{\sqrt{3}}\right) = \sqrt{\frac{3}{4}}$$

$$f \circ g\left(\frac{\pi}{4}\right) = \left(\frac{\sqrt{3}}{2}\right) \quad \checkmark$$

۲- الف

$$g\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{1-\sqrt{2}} \quad f\left(\frac{\sqrt{2}}{1-\sqrt{2}}\right) \rightarrow \left[\frac{\sqrt{2}}{1-\sqrt{2}}\right] = [-2.414] = (-4) \quad \textcircled{2}$$

$$f\left(\frac{\pi}{4}\right) = f(45^\circ) = \sin^2 45^\circ \rightarrow \frac{\sqrt{2}}{2} \quad \checkmark$$

$$g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \times \sqrt{1 - \frac{1}{2}} \rightarrow \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2} \quad \textcircled{2} \quad g \circ f\left(\frac{\pi}{4}\right) = \frac{1}{2}$$

۱۴- الف) $f \circ g(x) = 4 \rightarrow 6 \rightarrow 8$
 $2 \rightarrow 4 \rightarrow 8$
 $6 \rightarrow 8 \rightarrow 12$
 $8 \rightarrow 10 \rightarrow 12$

باید به اضافی مولفه های اول یکیا
 مولفه های دوم یکیا باشند پس
 تابع نیست.

ب) $g \circ f(x)$

$4 \rightarrow 8 \rightarrow x$
 $6 \rightarrow 8 \rightarrow x$
 $8 \rightarrow 12 \rightarrow x$
 $10 \rightarrow 12 \rightarrow x$

$\emptyset \quad \{ \}$ $\checkmark$

ب) $f \circ f(x)$

$4 \rightarrow 8 \rightarrow x$
 $6 \rightarrow 8 \rightarrow x$
 $8 \rightarrow 12 \rightarrow x$
 $10 \rightarrow 12 \rightarrow x$

$\emptyset \quad \{ \}$ $\checkmark$ $\textcircled{2}$

ج) $g \circ g(x) = 4 \rightarrow 6 \rightarrow 8 \quad \{(4, 8), (6, 6), (8, 10)\} \quad \checkmark$
 $2 \rightarrow 4 \rightarrow 6$
 $6 \rightarrow 8 \rightarrow 10$
 $8 \rightarrow 10 \rightarrow x$

تابع است

$$\begin{aligned} \text{fog}(x) &\rightarrow (x, y) \rightarrow a \rightarrow y \rightarrow x \Rightarrow \boxed{a=x} \\ \text{gof}(x) &= (x, 1) \rightarrow x \rightarrow d \rightarrow 1 \Rightarrow \boxed{b=d} \quad \boxed{(x, d)} \quad \text{✓} \end{aligned}$$

$$\begin{aligned} f(f(x)) &= (x+3) \rightarrow f(x) = x+1 \quad \underline{f(x) = -x-3} \\ f(-1) &= -1+1 = 0 \rightarrow g(-1) = 3(-1)-2 \rightarrow \boxed{-5} \quad \text{✓} \\ x+3 &= -1 \rightarrow x = -4 \end{aligned}$$

$$\begin{aligned} x \in \mathbb{R} \rightarrow x+|x| \geq 0 \rightarrow |x| \geq -x \rightarrow D_f = \mathbb{R} \cup [0, +\infty) \\ g(x) \xrightarrow{D_f = [0, +\infty)} \xrightarrow{D_f = \mathbb{R}} D(g \circ f) = [0, +\infty) \\ D_{g \circ f} = \{x \in D_f, f(x) \in D_g\} \quad D_{g \circ f} = (0, +\infty) - \{x\} \end{aligned}$$

$$\begin{aligned} f(x) &= \sqrt{(1-x)(1+x)} \quad f(x) \in D_{f+g} \\ g(x) &= \frac{-1}{x+1} \quad D_f = [-1, 1] \\ f+g(x) &\rightarrow [0, +\infty) \cap [-1, 1] = [0, 1] \\ D_{(f+g) \circ f} &= \{x \in D_f, f(x) \in D_{f+g}\} \rightarrow D_f \cap D_{f+g} \rightarrow [-1, 1] \end{aligned}$$

$$f\left(\frac{3x+1}{x-2}\right) = f(x+1) \rightarrow \text{نقطة} \rightarrow x = \frac{3+1}{1-2} \quad (9-الف)$$

$$f(t) = \frac{f(3+1)}{1-2} + \frac{3+1-1}{1-2} = \frac{13+1}{1-2} \rightarrow f(x) = \frac{13x-11}{x-2} \quad \checkmark$$

$$f\left(x + \frac{1}{x}\right) = x^3 + \frac{1}{x^3}$$

$$x + \frac{1}{x} = t \rightarrow \left(x + \frac{1}{x}\right)^3 = x^3 + 3x^2 \cdot \frac{1}{x} + 3x \cdot \frac{1}{x^2} + \frac{1}{x^3}$$

$$x^3 + 3x + \frac{3}{x} + \frac{1}{x^3} \rightarrow x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$$

$$x + \frac{1}{x} = t, f(t) = t^3 - 3t \rightarrow f(x) = x^3 - 3x \quad \checkmark$$

$$(x-1)(x-\sqrt{1}) \rightarrow x^2 - \sqrt{1}x - x + \sqrt{1}$$

$$(1,0)(\sqrt{1},0) \rightarrow f(x) = x \times \sqrt{x} \rightarrow \sqrt{1} \rightarrow 0 \rightarrow 0 \quad (\sqrt{1},0)$$

$$(\sqrt{1}-1) \quad \text{اختلاف طول نقاط}$$

$$1 \rightarrow 0 \rightarrow 0 \quad (1,0)$$

$$g(1) = 0 \rightarrow f(1) = 0 \rightarrow x\sqrt{x} = 1 \rightarrow x_1 = 1$$

$$g(\sqrt{x}) = 0 \rightarrow f(x) = 1\sqrt{x} \rightarrow x\sqrt{x} = \sqrt{x} \rightarrow x_2 = 1$$

$$x_2 - x_1 = 1 - 1 = 0$$