

$$f \circ f\left(\frac{\pi}{3}\right) \rightarrow \frac{\pi}{3} \times \frac{1}{4} \times \pi = \frac{\pi}{6} = 30^\circ \rightarrow 60 + 30 = 90^\circ$$

$$f(\sqrt{3}) \Rightarrow \sqrt{(\sqrt{3})^2 + 1} = 2$$

الف) $2 \cos \frac{\pi}{3} = \frac{1}{2} \Rightarrow f\left(\frac{1}{2}\right) = \sqrt{3}$

$$\Rightarrow \frac{\sqrt{3}}{2}$$

ب) $g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \rightarrow \frac{\sqrt{2}+2}{1-2} = \frac{\sqrt{2}+2}{-1}$

$$\Rightarrow -(\sqrt{2}+2)$$

$$f(-\sqrt{2}-2) = [-\sqrt{2}-2] \rightarrow [-\sqrt{2}]-2 = -2-2 = -4$$

$$f\left(\frac{\pi}{4}\right) \Rightarrow \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} \Rightarrow g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \times \sqrt{1-\frac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$

الف) $f \circ g(x) = \{(4, 5), (2, 5), (6, 12), (8, 12)\}$ ب) $g \circ f(x) = \emptyset$

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د) $g \circ g(x) = \{(4, 1), (2, 4), (4, 1)\}$

$$(2, 2) \in f \circ g \rightarrow \left. \begin{array}{l} 2 \in R_f \\ 2 \in D_g \\ a=2 \end{array} \right\} \rightarrow (a, 2) \Rightarrow (2, 2) \Rightarrow a=2$$

$$(2, 2) \rightarrow (4, 5)$$

$$(4, 1) \in g \circ f \rightarrow \left. \begin{array}{l} 1 \in R_g \\ 4 \in D_f \end{array} \right\} \rightarrow (4, 1) \rightarrow (b, 1) \Rightarrow b=4$$

$$f \circ f = a(ax+b)+b \Rightarrow ax+ab+b = cx+d$$

$$a^2 = c \Rightarrow a = \pm \sqrt{c} \quad ab+b=c \rightarrow b=1 \Rightarrow b=-c$$

$$a=\sqrt{c}, b=1 \Rightarrow f(x) = \sqrt{c}x+1$$

$$a=-\sqrt{c}, b=-c \Rightarrow f(x) = -\sqrt{c}x-c \rightarrow f(-1) = -1$$

$$g(x) = cx+d \Rightarrow c(\sqrt{c}x+1)+d = \sqrt{c}x-c$$

$$\sqrt{c}cx + \sqrt{c}c + d = \sqrt{c}x - c$$

$$\sqrt{c}cx = -1$$

$$\sqrt{c}x = -\frac{1}{\sqrt{c}} \Rightarrow x = -\frac{1}{c}$$

$$\text{if } x = -\frac{1}{c} \Rightarrow g(-1) = -1$$

$$\begin{cases} \sqrt{c}c+d = -1 \Rightarrow \frac{1}{\sqrt{c}}+d = -1 \Rightarrow d = -\frac{1}{\sqrt{c}}-1 \\ \sqrt{c}c = -c \Rightarrow c = -\frac{1}{\sqrt{c}} \end{cases} \Rightarrow g(x) = \frac{1}{\sqrt{c}}x - \frac{1}{\sqrt{c}} - 1$$

$$\rightarrow g(-1) = \frac{-1}{\sqrt{c}} - \frac{1}{\sqrt{c}} - 1 = -1 \Rightarrow g \circ f(-1) = -1$$

$$Dg \circ f = \{x \in D_f : f(x) \in Dg\} \quad Df = \mathbb{R}$$

$$x^2 - \varepsilon x = x(x - \varepsilon) \Rightarrow x \in \left[\frac{\varepsilon}{2}, \varepsilon \right] \Rightarrow Dg = \mathbb{R} - \{0, \varepsilon\} \rightarrow Dg \circ f = \{x \in \mathbb{R} : \sqrt{x+|x|} \neq 0, \varepsilon\}$$

$$x > 0, x \neq 1 \Rightarrow Dg \circ f = (0, +\infty) - \{1\}$$

$$(F+g) \circ f = F(F(x)) + G(F(x))$$

$$Df \Rightarrow x \leq 1$$

$$\sqrt{1-x^2} \leq 1$$

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$$F(F(x)) \Rightarrow \sqrt{1-(\sqrt{1-x^2})^2} = \sqrt{x^2} = |x|$$

$$G(F(x)) = \sqrt{\sqrt{1-x^2}} = \sqrt{1-x^2}$$

$$(F+g) \circ f = |x| + \sqrt{1-x^2}$$

$$1-x^2 \geq 0 \quad -1 \leq x \leq 1$$

$$[-1, 1]$$

$$f\left(\frac{x+1}{x-1}\right) \Rightarrow t = \frac{x+1}{x-1} \quad t(x-1) = x+1$$

(c)

$$tx - t = x + 1 \Rightarrow tx - x = t + 1$$

$$\Rightarrow x(t-1) = t+1 \Rightarrow x = \frac{t+1}{t-1}$$

$$F(t) = f(x+1) \Rightarrow \frac{1+t+1}{t-1} + 1$$

$$\Rightarrow \frac{1+t+1}{t-1} + 1 = \frac{1+t-1}{t-1} \Rightarrow f(x) = \frac{1+t-1}{t-1}$$

$$f\left(x + \frac{1}{x}\right) = x^3 + \frac{1}{x^3}$$

(c)

$$f\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$$

$$f(x) = x^3 - 3x$$

$$g \circ f = 0 \quad f(x) = 1 \rightarrow \frac{x\sqrt{x}}{x-1} = 1$$

$$f(x) = x\sqrt{x} \rightarrow \frac{x\sqrt{x}}{x-1} = 1$$

$$x-1=1$$