نام و نام خانوادگی ..پارسا جهانگیری...... پاسخنامه تشریحی تکلیف شماره ..۹.... کلاس یازدهم ریاضی A......

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$$fof(\tfrac{2}{3}) = f(f(\tfrac{2}{3})) = f(\sqrt{3}) = 2 \Rightarrow \boxed{fof(\tfrac{2}{3}) = 2}$$

$$f(x) = \begin{cases} \cot \frac{\pi x}{4} & ; x \le 1 \\ \sqrt{x^2+1} & ; x > 1 \end{cases} \quad \overset{\frac{2}{3}\le 1}{\Longrightarrow} f(\tfrac{2}{3}) = \cot\frac{\pi}{6} = \sqrt{3}, \quad \underset{\sqrt{3}>1}{\Longrightarrow} f(\sqrt{3}) = \sqrt{4} = 2$$

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الف) $fog(\frac{\pi}{3}) = f(g(\frac{\pi}{3})) = f(\frac{1}{2}) = \frac{\sqrt{3}}{2}$ $\quad \boxed{fog(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}}$

$$g(x) = 2\cos^2 x \overset{x=\frac{\pi}{3}}{\Longrightarrow} g(\tfrac{\pi}{3}) = 2\cos^2\frac{\pi}{3} = 2(\tfrac{1}{2})^2 = \tfrac{1}{2}$$

$$f(\tfrac{1}{x}) = \sqrt{\frac{2x-1}{x^2}} \overset{\frac{1}{x}=\frac{1}{2} \Rightarrow x=2}{\Longrightarrow} f(\tfrac{1}{2}) = \sqrt{\tfrac{3}{4}} = \tfrac{\sqrt{3}}{2}$$

ب) $fog(\sqrt{2}) = f(g(\sqrt{2})) = f(-2-\sqrt{2}) = -4$ $\quad \boxed{fog(\sqrt{2}) = -4}$

$$g(x) = \frac{x}{1-x} \overset{x=\sqrt{2}}{\Longrightarrow} g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} = \frac{\sqrt{2}(1+\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})} = -(2+\sqrt{2}) = -2-\sqrt{2}$$

$$f(x) = [x] \overset{x=-2-\sqrt{2}}{\Longrightarrow} f(-2-\sqrt{2}) = [-2-\sqrt{2}] \underset{\sqrt{2}=1.4}{=} [-3.4] = -4$$

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$gof(\frac{\pi}{4}) = g(f(\frac{\pi}{4})) = g(\frac{\sqrt{2}}{2}) = \frac{1}{2}$ $\quad \boxed{gof(\frac{\pi}{4}) = \frac{1}{2}}$

$$f(x) = \sin x \overset{x=\frac{\pi}{4}}{\Longrightarrow} f(\tfrac{\pi}{4}) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$g(x) = x\sqrt{1-x^2} \overset{x=\frac{\sqrt{2}}{2}}{\Longrightarrow} g(\tfrac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2}\sqrt{1-\tfrac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$

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الف) $fog(x) = \{(4,5), (2,5), (6,12), (8,12)\}$

$4 \to 6 \to 5$
$2 \to 6 \to 5$
$6 \to 8 \to 12$
$8 \to 10 \to 12$

ب) $gof(x) = \emptyset$

$4 \to 5 \to \times$
$6 \to 5 \to \times$
$8 \to 12 \to \times$
$10 \to 12 \to \times$

ج) $fof(x) = \emptyset$

$4 \to 5 \to \times$
$6 \to 5 \to \times$
$8 \to 12 \to \times$
$10 \to 12 \to \times$

د) $gog(x) = \{(4,8), (2,6), (6,10)\}$

$4 \to 6 \to 8$
$2 \to 4 \to 6$
$6 \to 8 \to 10$
$8 \to 10 \to \times$

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$\boxed{(4,2) \in fog}$ (I)

$1 \to 2 \to 1$
$3 \to 1 \to 7$
$a \to 3 \to 2$
$b \to 1 \to 7$

$fog(x) = \{(1,1), (3,7), (a,2), (b,7)\}$ (II)

$\overset{(I),(II)}{\Longrightarrow} (4,2) = (a,2) \Rightarrow \boxed{a=4}$ (V)

$(4,1) \in gof$

$2 \to 1 \to 2$
$3 \to 2 \to 3$
$4 \to b \to 1$ (III)
$1 \to 7$

$f = \{(2,1), (3,2), (4,5), (1,7)\}$ (IV)

$\overset{(III),(IV)}{\Longrightarrow} (4,b) = (4,5) \Rightarrow \boxed{b=5}$ (VI)

$\overset{(V),(VI)}{\Longrightarrow} (a,b) = (4,5)$

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f تابعی خطی است پس آن را به فرم $f(x)=ax+b$ می نویسیم.

$f(f(x))=f(ax+b)=a^2x+ab+b=4x+3$ (متحد قرار می دهیم) $\Rightarrow \begin{cases} a^2=4 \Rightarrow a=\pm 2 \\ ab+b=3 \end{cases}$

$\xrightarrow{a=2} 3b=3 \Rightarrow \boxed{b=1}$ ، $\xrightarrow{a=-2} -b=3 \Rightarrow \boxed{b=-3}$

$f(x)=ax+b \xrightarrow{a=2,\ b=1} f(x)=2x+1 \xrightarrow{x=-1} f(-1)=-1$

$\xrightarrow{a=-2,\ b=-3} f(x)=-2x-3 \xrightarrow{x=-1} f(-1)=-1$

$\Rightarrow \boxed{f(-1)=-1}$

$gof(-1)=g(f(-1))=g(-1)=-8$　　$\boxed{gof(-1)=-8}$

$g(2x+3)=3x-2 \xrightarrow[x=-2]{2x+3=-1} g(-1)=-8$

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$D_{gof}=\{x \mid x\in D_f \ ;\ f(x)\in D_g\}$

$f(x)=\sqrt{x+|x|} \Rightarrow x+|x|\geq 0 \Rightarrow |x|\geq -x$ همواره برقرار است $\Rightarrow \boxed{D_f=\mathbb{R}}$

$g(x)=\dfrac{1}{x^2-4x} \Rightarrow x^2-4x\neq 0 \Rightarrow x(x-4)\neq 0 \Rightarrow x\neq 0,4 \Rightarrow \boxed{D_g=\mathbb{R}-\{0,4\}}$

$\boxed{x\in\mathbb{R}}$ (I)

$\begin{cases} \sqrt{x+|x|}\neq 0 \Rightarrow x+|x|\neq 0 \Rightarrow |x|\neq -x \Rightarrow \boxed{x>0} \ \text{(II)} \\ \sqrt{x+|x|}\neq 4 \Rightarrow x+|x|\neq 16 \xrightarrow{x\geq 0} 2x\neq 16 \Rightarrow \boxed{x\neq 8} \ \text{(III)} \end{cases}$

(I)∩(II)∩(III) $\Rightarrow \boxed{D_{gof}=(0,+\infty)-\{8\}}$

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$D_{(f+g)of}=\{x \mid x\in D_f \ ;\ f(x)\in D_{f+g}\}$

$D_{f+g}=D_f\cap D_g \xrightarrow{(I),(II)} D_{f+g}=[0,1]$

$g(x)=\sqrt{x} \Rightarrow x\geq 0 \Rightarrow D_g=[0,+\infty)$ (I)

$f(x)=\sqrt{1-x^2} \Rightarrow 1-x^2\geq 0 \Rightarrow (1-x)(1+x)\geq 0$

$-1 \quad 1 \rightarrow D_f=[-1,1]$ (II)

$\boxed{x\in[-1,1]}$ (III)

$\begin{cases} \sqrt{1-x^2}\in[0,1] \\ \Rightarrow \sqrt{1-x^2}\leq 1 \Rightarrow 1-x^2\leq 1 \\ \Rightarrow x^2\geq 0 \Rightarrow \boxed{x\in\mathbb{R}} \ \text{(IV)} \end{cases}$ همواره برقرار است

(III)∩(IV) $\Rightarrow \boxed{D_{(f+g)of}=[0,1]}$

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الف) $f\left(\dfrac{3x+1}{x-2}\right)=4x+5$

$\dfrac{3x+1}{x-2}=t \Rightarrow \dfrac{2t+1}{t-3}=x$

$\Rightarrow f(t)=4\left(\dfrac{2t+1}{t-3}\right)+5=\dfrac{8t+4}{t-3}+5=\dfrac{13t-11}{t-3} \Rightarrow \boxed{f(x)=\dfrac{13x-11}{x-3}}$

ب) $f\left(x+\dfrac{1}{x}\right)=x^3+\dfrac{1}{x^3} \xrightarrow{x+\frac{1}{x}=t} f(t)=t^3-3t \Rightarrow \boxed{f(x)=x^3-3x}$

$x^3+\dfrac{1}{x^3}=\left(x+\dfrac{1}{x}\right)^3-3(x)\left(\dfrac{1}{x}\right)\left(x+\dfrac{1}{x}\right) \Rightarrow x^3+\dfrac{1}{x^3}=\left(x+\dfrac{1}{x}\right)^3-3\left(x+\dfrac{1}{x}\right)$

$\xrightarrow{x+\frac{1}{x}=t} x^3+\dfrac{1}{x^3}=t^3-3t$

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$\left.\begin{array}{l} g(f(x))=0 \\ g(1)=0 \\ g(2\sqrt{2})=0 \end{array}\right\} \Rightarrow \begin{cases} f(x)=1=x\sqrt{x} \xrightarrow{()^2} x^3=1 \Rightarrow \boxed{x=1} \\ f(x)=2\sqrt{2}=x\sqrt{x} \xrightarrow{()^2} x^3=8 \Rightarrow \boxed{x=2} \end{cases}$

$\Rightarrow 2-1=①$