

۱	$f\left(\frac{\pi}{3}\right) = \cot \frac{\pi}{3} = \cot \frac{\pi}{4} = \sqrt{3}$ $f(\sqrt{3}) = \sqrt{(\sqrt{3})^2 + 1} = 2$
۲	الف) $g\left(\frac{\pi}{3}\right) = 2 \cos^2 \frac{\pi}{3} = 2 \left(\frac{1}{2}\right)^2 = \frac{1}{2}$ $f\left(\frac{1}{2}\right) = \sqrt{\frac{2(2)-1}{2}} = \frac{\sqrt{3}}{2}$ ب) $g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \xrightarrow{\sqrt{2} \approx 1.4} f\left(\frac{\sqrt{2}}{1-\sqrt{2}}\right) = \left[\frac{1/4}{1-1.4}\right] = -4$
۳	$f\left(\frac{\pi}{4}\right) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\rightarrow g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \sqrt{1 - \left(\frac{\sqrt{2}}{2}\right)^2} = \frac{1}{2}$
۴	الف) $f \circ g(x) = \{(1, 5), (2, 5), (4, 12), (8, 12)\}$ ب) $g \circ f(x) = \{0\}$ ج) $f \circ f(x) = \{0\}$ د) $g \circ g(x) = \{(1, 8), (2, 4), (4, 0)\}$
۵	$\left. \begin{array}{l} f(g(1)) = 2 \rightarrow f(3) = 2 \rightarrow g(2) = 3 \rightarrow a = 2 \\ g(f(1)) = 1 \rightarrow f(1) = 5 \rightarrow g(5) = 1 \rightarrow b = 5 \end{array} \right\} (a, b) = (2, 5)$

$$f(x) = ax + b \rightarrow f \circ f(x) = a^2x + b(a+1) = tx + r \rightarrow a = \pm r, b = 1, -r$$

$$f(x) = rx + 1, -rx - r \quad \checkmark$$

$$rx + r = t \rightarrow x = \frac{t-r}{r} \rightarrow g(t) = \frac{r+t-r}{r}$$

$$f(-1) = -1 \rightarrow g(-1) = -1 \quad \checkmark$$

$$x + |x| > 0 \rightarrow D_f \in \mathbb{R} \rightarrow R_f = [0, +\infty)$$

$$g(x) = \frac{1}{x^2 - tx} \rightarrow x^2 - tx \neq 0 \rightarrow x \neq 0, t \rightarrow R_f \neq 0, t$$

$$\rightarrow D_f \neq (-\infty, 0] \cup \{1\} \Rightarrow D_f \in \mathbb{R} - (-\infty, 0] \cup \{1\} \quad \checkmark$$

$$D_f = (0, +\infty) - \{1\}$$

$$f(x) = \sqrt{1-x^2} \rightarrow 1-x^2 > 0 \rightarrow D_f = [-1, 1] \rightarrow R_f = [0, 1]$$

$$D_{f+g} = [0, 1]$$

$$D_{f+g} = \{x \in D_f \cap f(x) \in D_{f+g}\} = [-1, 1] \quad \checkmark$$

$$\text{الف) } \frac{rx+1}{x-r} = t \rightarrow x = \frac{t+1}{t-r} \rightarrow tx + a = \frac{1+t+t+at-1a}{t-r} = \frac{1r+t-11}{t-r}$$

$$\rightarrow f(x) = \frac{1rx-11}{x-r} \quad \checkmark$$

$$\rightarrow f(x + \frac{1}{x}) = (x + \frac{1}{x})^r - r(x + \frac{1}{x}) \rightarrow f(x) = x^r - rx \quad \checkmark$$

$$g(x) = 0 \rightarrow x = r\sqrt{r+1}$$

$$g(f(x)) = 0 \rightarrow f(x) = 1, r\sqrt{r} \rightarrow x\sqrt{x} = 1 \Rightarrow x = 1 \quad \checkmark$$

$$x\sqrt{x} = r\sqrt{r} \Rightarrow x = r \quad \checkmark$$

$$\rightarrow x_1 - x_r = r - 1 = 1 \quad \checkmark$$