نام و نام خانوادگی ....علی صفری.... پاسخنامه تشریحی تکلیف شماره ۹ ..... کلاس یازدهم تجربی A

۱

$$f_{(x)}=\begin{cases}\cot\frac{\pi x}{4} & ; x\leq 1\\ \sqrt{x^2+1} & ; x>1\end{cases} \rightarrow f\circ f_{(\frac{2}{3})}=f\Big(\underbrace{f_{(\frac{2}{3})}}_{\cot\frac{\pi}{6}}\Big)=f_{(\sqrt{3})}$$

$$\rightarrow f_{(\sqrt{3})}=\sqrt{3+1}=2$$

۲

الف)

$$f_{(\frac{1}{x})}=\sqrt{\frac{2x-1}{x^2}}\ ,\ g_{(x)}=2\cos^2 x \rightarrow f\circ g\,(\tfrac{\pi}{3})=f\,(\tfrac{1}{2})$$

$$\xrightarrow{x=2} f_{(\frac{1}{2})}=\sqrt{\frac{2\times 2-1}{4}}=\sqrt{\frac{3}{4}}$$

ب)

$$g(\sqrt{2})=\frac{\sqrt{2}}{1-\sqrt{2}} \rightarrow f\circ g\,(\sqrt{2})=\left[\frac{\sqrt{2}}{1-\sqrt{2}}\right]=-4$$

۳

$$f_{(x)}=\sin x\ ,\ g_{(x)}=x\sqrt{1-x^2} \rightarrow g\circ f\,(\tfrac{\pi}{4})=g\,(f_{(\frac{\pi}{4})})$$

$$=g\,(\sin\tfrac{\pi}{4})=g\,(\tfrac{\sqrt{2}}{2})=\frac{\sqrt{2}}{2}\sqrt{1-\frac{1}{2}}=\frac{\sqrt{2}}{2}\times\frac{1}{\sqrt{2}}=\frac{1}{2}$$

۴

$$f_{(x)}=\{(4,5),(6,5),(8,12),(10,12)\}\qquad g_{(x)}=\{(4,6),(2,4),(6,8),(8,10)\}$$

الف) $f\circ g\,(x) \rightarrow \{(4,5),(2,5),(6,12),(8,12)\}$

ب) $g\circ f_{(x)}=\varnothing$

د) $g\circ g_{(x)}=\{(4,8),(2,6),(6,10)\}$

ج) $f\circ f_{(x)}=\varnothing$

۵

$$f=\{(2,1),(3,2),(4,5),(1,7)\}\qquad g=\{(1,2),(3,1),(\underset{4}{a},3),(\underset{5}{b},1)\}$$

$(3,2)\in f\circ g \rightarrow a\rightarrow 3\rightarrow 2 \rightarrow a=4$

$(4,1)\in g\circ f \rightarrow 4\rightarrow 5=b\rightarrow 1$

$\rightarrow (a,b)=(4,5)$

$$f(f_{(x)}) = 4x+3 \rightarrow a^2x + ab + b = 4x+3 \rightarrow \begin{cases} a=2 \rightarrow b=1 \rightarrow f_{(x)} = 2x+1 \\ a=-2 \rightarrow b=-3 \rightarrow f_{(x)} = -2x-3 \end{cases}$$

$$\rightarrow f_{(-1)} = -1$$

$$g_{(2x+3)} = 3x-2 \rightarrow 2x+3 = t \rightarrow x = \frac{t-3}{2} \rightarrow g_{(x)} = \frac{3}{2}x - \frac{9}{2} - 2$$ ۶

$$\rightarrow x=-1 \rightarrow g_{(x)} = -8 \qquad \rightarrow g\circ f_{(-1)} = -8$$

$$f_{(x)} = \sqrt{x+|x|} \qquad g_{(x)} = \frac{1}{x^2-4x} \qquad D_{g\circ f} = ?$$

$$D_{g\circ f} = \{x \in D_f \ , \ f_{(x)} \in D_g\} = \{x \in \mathbb{R} \wedge f_{(x)} \in \mathbb{R} - \{0,4\}\}$$ ۷

$$= \{x \in \mathbb{R} \wedge (x>0 \quad x \neq 8)\} \rightarrow D_{g\circ f} = (0,+\infty) - \{8\}$$

$$f_{(x)} = \sqrt{1-x^2} \qquad g_{(x)} = \sqrt{x} \rightarrow D_{(f+g)\circ f} = ?$$

$$D_{(f+g)\circ f} = \{x \in D_f \ , \ f_{(x)} \in D_{(f+g)}\} \rightarrow \{-1 \leq x \leq 1 \wedge 0 \leq \sqrt{1-x^2} \leq 1\}$$ ۸

$$\{-1 \leq x \leq 1 \wedge \sqrt{1-x^2} \leq 1\} \Rightarrow \{-1 \leq x \leq 1 \wedge 0 \leq 1-x^2 \leq 1\} = \{-1 \leq x \leq 1 \wedge 0 \leq x^2 \leq 1\}$$

$$= \{-1 \leq x \leq 1 \wedge -1 \leq x \leq 1\} \rightarrow D_{(f+g)\circ f} = [-1,1]$$

$$f\left(\frac{3x+1}{x-2}\right) = 4x+5 \rightarrow \frac{3x+1}{x-1} = t \rightarrow x = \frac{2t+1}{t-3} \rightarrow f_{(t)} = \frac{8t+4+5t-15}{t-3}$$

$$f_{(x)} = \frac{13x-11}{x-3}$$ ۹

$$f\left(x+\frac{1}{x}\right) = x^3 + \frac{1}{x^3} \rightarrow x+\frac{1}{x} = t \rightarrow f\left(x+\frac{1}{x}\right) = \left(x+\frac{1}{x}\right)^3 - 3\left(x \times \frac{1}{x}\right)\left(x+\frac{1}{x}\right)$$

$$\rightarrow f_{(t)} = t^3 - 3t \rightarrow f_{(x)} = x^3 - 3x$$

$$f_{(x)} = x\sqrt{x} \qquad \begin{cases} g_{(1)} = 0 \\ g_{(2\sqrt{2})} = 0 \end{cases} \rightarrow g\circ f_{(x)} = 0$$

$$\left.\begin{array}{l} \rightarrow f_{(x)} = 1 \rightarrow x = 1 \\ \rightarrow f_{(x)} = 2\sqrt{2} \rightarrow x = 2 \end{array}\right\} \quad 2-1 = 1$$ ۱۰