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9. $f: A \rightarrow B$, $g: B \rightarrow C$, $h: C \rightarrow D$

$$\begin{cases} \cot \frac{\pi x}{r} & x \leq 1 \\ \sqrt{x^2+1} & x > 1 \end{cases} \rightarrow f(f(\frac{r}{r})) = f(1) = r \Rightarrow f \circ f(\frac{r}{r}) = r$$

$$\cot \frac{\pi}{r} = \sqrt{r}$$

$$f(r) = \sqrt{r} = r$$

$$f(\frac{1}{x}) = \sqrt{\frac{r x - 1}{x^2}} , g(x) = r \cos^2 x \quad f \circ g(\frac{\pi}{r})$$

$$f(g(\frac{\pi}{r})) = f(\frac{1}{r}) = \frac{\sqrt{r}}{r} \quad g(x) = r \cos^2 x \rightarrow g(\frac{\pi}{r}) = r(\frac{1}{r})^2 = \frac{1}{r}$$

$$\sqrt{\frac{r x - 1}{x^2}} \rightarrow \frac{1}{x} = \frac{1}{r} \rightarrow x = r \rightarrow \sqrt{\frac{r}{r}} = \frac{\sqrt{r}}{r}$$

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$$f(x) = [x] \quad g(x) = \frac{x}{1-x} \quad f \circ g(\sqrt{r}) = 9$$

$$f(g(\sqrt{r})) = f(-r - \sqrt{r}) = -9$$

$$g(x) = \frac{x}{1-x} \rightarrow \frac{\sqrt{r}}{\sqrt{r}-1} = -r - \sqrt{r} = -(r + \sqrt{r})$$

$$15 \quad f(x) = [x] \Rightarrow f(-r - \sqrt{r}) = [-r - \sqrt{r}] = -9$$

$$f(x) = \sin x \quad g(x) = x \sqrt{1-x^2} \quad g \circ f(\frac{\pi}{2})$$

$$g \circ f(\frac{\pi}{2}) = g(f(\frac{\pi}{2})) = g(\frac{\sqrt{r}}{r}) = \frac{1}{r}$$

$$f(\frac{\pi}{2}) = \sin \frac{\pi}{2} = \frac{\sqrt{r}}{r}$$

$$20 \quad g(\frac{\sqrt{r}}{r}) = \frac{\sqrt{r}}{r} \sqrt{1 - \frac{1}{r}} = \frac{1}{r}$$

$$f(x) = \{(r, \omega)(y, \omega)(\wedge, 1r)(1., 1r)\} \quad f \circ g(x) =$$

$$\begin{matrix} r \rightarrow \omega \rightarrow \omega \\ y \rightarrow \omega \rightarrow \omega \\ \wedge \rightarrow 1r \rightarrow 1r \\ 1. \rightarrow 1r \rightarrow 1r \end{matrix} = \{(r, \omega)(y, \omega)(\wedge, 1r)(1., 1r)\}$$

$$g(x) = \{(r, y)(r, \varepsilon)(y, \wedge)(\wedge, 1.)\}$$

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$$g \circ f(x) =$$

$$\begin{matrix} r \rightarrow \omega \rightarrow x \\ y \rightarrow \omega \rightarrow x \\ \wedge \rightarrow 1r \rightarrow x \\ 1. \rightarrow 1r \rightarrow x \end{matrix} = \emptyset$$

$$f \circ f(x) = \begin{matrix} r \rightarrow \omega \rightarrow x \\ y \rightarrow \omega \rightarrow x \\ \wedge \rightarrow 1r \rightarrow x \\ 1. \rightarrow 1r \rightarrow x \end{matrix} = \emptyset$$

$$g \circ g(x) =$$

$$\begin{matrix} r \rightarrow y \rightarrow \wedge \\ y \rightarrow \wedge \rightarrow 1. \\ \wedge \rightarrow 1. \rightarrow x \end{matrix} = \{(r, \wedge)(y, y)(y, 1.)\}$$

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$$f = \{(r, 1), (r, r), (r, a), (1, v)\}$$

$$g = \{(1, r), (r, 1), (a, r), (b, 1)\}$$

$$1 \rightarrow r \rightarrow 1$$

$$r \rightarrow 1 \rightarrow v$$

$$a \rightarrow r \rightarrow r$$

$$b \rightarrow 1 \rightarrow v$$

$$f \circ g \{(1, 1), (r, v), (a, r), (b, v)\}$$

$$(r, r) = (a, r) \rightarrow a = r$$

$$s \quad (a, b) = (r, a)$$

$$r \rightarrow 1 \rightarrow r$$

$$r \rightarrow r \rightarrow r$$

$$r \rightarrow b$$

$$1 \rightarrow v$$

$$(r, b) = (r, a)$$

$$b = a$$

$$f(f(x)) = rx + r \rightarrow g \circ f(-1)$$

$$g(rx + r) = rx - r$$

$$f(ax + b) = arx + ab + b = rx + r$$

$$ar = r \rightarrow a = 1$$

$$ab + b = r \rightarrow a = r$$

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$$g \circ f(-1) = g(f(-1)) = g(-1) = -1$$

$$g(rx + r) = rx - r$$

$$x = -r$$

$$f(x) = ax + b \rightarrow rx + 1 \rightarrow f(-1) = -1$$

$$-1x + r \rightarrow f(-1) = -1$$

$$rb = r$$

$$b = 1$$

$$a = -r$$

$$b = -r$$

$$g(-1) = -1$$

$$g \circ f(-1) = -1$$

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$$f(x) = \sqrt{x+|x|}$$

$$g(x) = \frac{1}{x^2 - rx} \rightarrow g \circ f$$

$$x + |x| \geq 0 \rightarrow \sqrt{}$$

$$Df = \mathbb{R}$$

$$\frac{1}{x^2 - rx} \Rightarrow x^2 - rx \neq 0 \rightarrow x \neq 0, r$$

$$x + |x| \neq 0 \rightarrow |x| \neq -x$$

$$x > 0$$

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$$x + |x| \neq 0 \rightarrow \frac{0}{x} \neq 0$$

$$Dg \circ f = (0, +\infty) - \{1\}$$

$$(0, 1) \cup (1, +\infty)$$

$$f(x) = \sqrt{1-x^2} \quad g(x) = \sqrt{x} \quad (f \circ g) \circ f$$

$$x \in Df \quad f(x) \in D_{f \circ g}$$

$$[-1, 1] \quad \sqrt{1-x^2} \in [0, 1]$$

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$$\sqrt{1-x^2} \leq 1 \rightarrow 1-x^2 \leq 1$$

$$x^2 \geq 0 \quad x \in \mathbb{R}$$

$$D_{f \circ g} = Df \cap Dg \Rightarrow [0, 1]$$

$$g(x) = \sqrt{x} \Rightarrow x \geq 0 \rightarrow Dg = [0, +\infty)$$

$$f(x) = \sqrt{1-x^2} \Rightarrow 1-x^2 \geq 0 \rightarrow (1-x)(1+x) \geq 0$$

$$[-1, 1]$$

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$$Df \cap Dg = [0, 1]$$

KHAJOO TAHRIR

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$$f\left(\frac{3x+1}{x-2}\right) = 4x+2$$

$$\frac{3x+1}{x-2} = t \Rightarrow \frac{3t+1}{t-2} = x$$

$$f(t) = 4\left(\frac{3t+1}{t-2}\right) + 2 = \frac{12t+14}{t-2}$$

$$\frac{12t-11}{t-2} \rightarrow f(x) = \frac{12x-11}{x-2}$$

5.

$$f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} \rightarrow x^2 + \frac{1}{x^2} = t \rightarrow f(t) = t^2 - 2t$$

$$f(x) = x^2 - 2x = \left(x + \frac{1}{x}\right)^2 - 2\left(x + \frac{1}{x}\right)\left(x + \frac{1}{x}\right)$$

$$x^2 + \frac{1}{x^2} = t^2 - 2t$$

10.

$$g(f(x)) = 0$$

$$g(1) = 0$$

$$g(\sqrt{x}) = 0$$

$$f(x) = 1 \Rightarrow x\sqrt{x} \rightarrow x^3 = 1 \Rightarrow x = 1$$

$$f(x) = \sqrt{x} \Rightarrow x\sqrt{x} \rightarrow x^3 = 8 \Rightarrow x = 2$$

$$x-1=1$$

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