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سوال 9 کثرت و کثرت در تابع

(سوال 1)

$$f(x) = \begin{cases} G\left(\frac{\pi x}{\sqrt{x^2+1}}\right) & x \leq 1 \\ \sqrt{x^2+1} & x > 1 \end{cases}$$

$$\begin{aligned} f \circ f\left(\frac{1}{\sqrt{2}}\right) &= ? \\ f\left(f\left(\frac{1}{\sqrt{2}}\right)\right) &= ? \\ \Rightarrow f\left(\frac{1}{\sqrt{2}}\right) &= G\left(\frac{\pi}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)\right) = G\left(\frac{\pi}{2}\right) = \sqrt{2} \end{aligned}$$

$$\begin{aligned} \frac{\sqrt{2} > 1}{\sqrt{2} > 1} \Rightarrow f(\sqrt{2}) &= \sqrt{(\sqrt{2})^2 + 1} = \sqrt{3} = \sqrt{3} \end{aligned}$$

$$\Rightarrow f(f(\frac{1}{\sqrt{2}})) = \sqrt{3} \quad \text{✓} \quad \textcircled{2}$$

سوال 12 الف

$$\begin{aligned} f \circ g\left(\frac{\pi}{\sqrt{2}}\right) &= ? \quad g(x) = 2 \cos^2 x, \quad f\left(\frac{1}{x}\right) = \sqrt{\frac{x^2-1}{x^2}} \\ f \circ g\left(\frac{\pi}{\sqrt{2}}\right) &= f\left(g\left(\frac{\pi}{\sqrt{2}}\right)\right) \Rightarrow g\left(\frac{\pi}{\sqrt{2}}\right) = 2 \cos^2 \frac{\pi}{\sqrt{2}} = 2 \left(\frac{1}{\sqrt{2}}\right)^2 = 2 \times \frac{1}{2} = 1 \end{aligned}$$

$$f\left(\frac{1}{1}\right) \Rightarrow \sqrt{\frac{1^2-1}{1^2}} = \sqrt{\frac{0}{1}} = \sqrt{0} = 0 \Rightarrow f \circ g\left(\frac{\pi}{\sqrt{2}}\right) = 0$$

$$\textcircled{1,5} \quad f \circ g(\sqrt{2})? \quad g(x) = \frac{x}{1-x}, \quad f(x) = [x] \quad \text{✓}$$

$$f \circ g(\sqrt{2}) = f(g(\sqrt{2})) \Rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}(1+\sqrt{2})}{-1} = \frac{\sqrt{2}+\sqrt{2}}{-1} = -\sqrt{2}-\sqrt{2} = -2\sqrt{2}$$

$$f(-2\sqrt{2}) = [-2\sqrt{2}] \Rightarrow \frac{\sqrt{2} \approx 1.41}{-2\sqrt{2} \approx -2.82} \Rightarrow [-2.82] = -3 \Rightarrow f \circ g(\sqrt{2}) = -3 \quad \text{✓}$$

$$g \circ f\left(\frac{\pi}{\sqrt{2}}\right) \quad \text{مقدار} \quad g(x) = x\sqrt{1-x^2}, \quad f(x) = \sin x \quad \text{سوال 13}$$

$$g \circ f\left(\frac{\pi}{\sqrt{2}}\right) = g\left(f\left(\frac{\pi}{\sqrt{2}}\right)\right) \Rightarrow f\left(\frac{\pi}{\sqrt{2}}\right) = \sin\left(\frac{\pi}{\sqrt{2}}\right) = \frac{\sqrt{2}}{\sqrt{2}} = 1 \quad \text{✓} \quad \textcircled{2}$$

$$\Rightarrow g\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{2}}{\sqrt{2}} \sqrt{1-\left(\frac{\sqrt{2}}{\sqrt{2}}\right)^2} = \frac{\sqrt{2}}{\sqrt{2}} \sqrt{1-\frac{1}{2}} = \frac{\sqrt{2}}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \quad g \circ f\left(\frac{\pi}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}} \quad \text{✓}$$

سوال 14

$$f(x) = \{(1, 2), (2, 3), (3, 4), (4, 5)\} \quad g(x) = \{(1, 2), (2, 3), (3, 4), (4, 5)\}$$

$$\begin{aligned} \text{الف) } f \circ g(x) &= f(g(x)) \\ f &\rightarrow 2 \rightarrow 3 \\ g &\rightarrow 1 \rightarrow 2 \\ 2 &\rightarrow 3 \rightarrow 4 \\ 3 &\rightarrow 4 \rightarrow 5 \\ 4 &\rightarrow 5 \rightarrow 6 \end{aligned} \Rightarrow \{(2, 3), (3, 4), (4, 5), (5, 6)\} \quad \text{✓}$$

$$\begin{aligned} \text{ب) } g \circ f(x) &= g(f(x)) \\ f &\rightarrow 1 \rightarrow 2 \\ g &\rightarrow 2 \rightarrow 3 \\ 1 &\rightarrow 2 \rightarrow 3 \\ 2 &\rightarrow 3 \rightarrow 4 \\ 3 &\rightarrow 4 \rightarrow 5 \\ 4 &\rightarrow 5 \rightarrow 6 \end{aligned} \Rightarrow \emptyset \quad \text{✓}$$

$$ج) f \circ f(x) = f(f(x))$$

$$\begin{aligned} f &\rightarrow \omega \rightarrow X \\ f &\rightarrow \omega \rightarrow X \\ \wedge &\rightarrow \omega \rightarrow X \\ 1 &\rightarrow \omega \rightarrow X \end{aligned}$$

$\Rightarrow \emptyset$ ✓

②

$$د) g \circ g(x) = g(g(x))$$

$$\begin{aligned} f &\rightarrow f \rightarrow 1 \\ f &\rightarrow f \rightarrow f \\ f &\rightarrow 1 \rightarrow 1 \\ \wedge &\rightarrow 1 \rightarrow X \end{aligned}$$

$$\Rightarrow \{(f, \wedge), (f, f), (f, 1)\}$$

$$f = \{(f, 1), (f, f), (f, \omega), (1, f)\}$$

$$g = \{(1, f), (f, 1), (a, f), (b, 1)\}$$

$$(f, f) \in f \circ g \Rightarrow (f, f) \in f(g(f))$$

$$f \leftarrow f \leftarrow f \Rightarrow a = f$$

$$(a, b) = (f, \omega)$$

$$(f, 1) \in g \circ f \Rightarrow (f, 1) \in g(f(f))$$

$$f \rightarrow \omega \rightarrow 1 \Rightarrow b = \omega$$

$$\begin{aligned} f(f(x)) &= fx + f \\ g(fx + f) &= fx - f \\ g \circ f(-1) &? \end{aligned}$$

$$\begin{aligned} a &= \pm f \Rightarrow \begin{cases} a = f, b = f \Rightarrow b = 1 \Rightarrow f(x) = fx + 1 \\ a = -f, b = f \Rightarrow b = -f \Rightarrow f(x) = -x - f \end{cases} \\ f \circ f &= f \Rightarrow f(f(x)) = fx + f \Rightarrow a(fx + f) + b = fx + f \Rightarrow afx + ab + b = fx + f \\ g \circ g &= g \Rightarrow g(g(x)) = fx - f \Rightarrow c(fx + f) + d = fx - f \Rightarrow cf + d = -f \end{aligned}$$

$$\begin{aligned} g \circ f(-1) &= g(f(-1)) \Rightarrow \begin{cases} g(f(-1) + 1) = g(-1) = \frac{f}{f}(-1) - \frac{1f}{f} = \frac{-1f}{f} = -1 \\ g(-(-1) - f) = g(-f) = \frac{f}{f}(-f) - \frac{1f}{f} = \frac{-1f}{f} = -1 \end{cases} \\ \Rightarrow g(-1) &= -1 \end{aligned}$$

①

$$f(x) = \sqrt{x+|x|}$$

$$g(x) = \frac{1}{x^2 - 1}$$

$$g \circ f \rightarrow D_{g \circ f} = ?$$

$$D_f = \mathbb{R}$$

$$D_g = \mathbb{R} - \{0, 1\}$$

$$D_{g \circ f} = \mathbb{R} - \mathbb{R}^{-1} \cup \{0\} \cup \{1\}$$

$$g \circ f(x) = \left\{ x \in D_f \mid f(x) \in D_g \right\}$$

(د سوال)

$$\Rightarrow x^2 - 1 \neq 0 \Rightarrow x(x-1) \neq 0 \Rightarrow \begin{matrix} x \neq 0 \\ x \neq 1 \end{matrix}$$

$$\sqrt{x+|x|} = 0 \Rightarrow x+|x| = 0 \Rightarrow \mathbb{R}^{-1} \cup \{0\}$$

$$\sqrt{x+|x|} = 1 \Rightarrow x+|x| = 1 \Rightarrow x=1 \Rightarrow x=1$$

از این شرط ها $a=1$ می آید
و اینها را باید قبول است.

$$f(x) \neq 0, 1 \rightarrow \begin{cases} \sqrt{x+|x|} \neq 0 \rightarrow x > 0 \\ \sqrt{x+|x|} \neq 1 \rightarrow x \neq 1 \end{cases}$$

$$\textcircled{1} \cap \textcircled{2} = (0, +\infty) - \{1\}$$

$$f(x) = \sqrt{1-x^2}$$

$$g(x) = \sqrt{x}$$

$$(f+g) \circ f = ?$$

$$(f+g) \circ f = f \circ f + g \circ f = f(f(x)) + g(f(x))$$

$$= \sqrt{1-\sqrt{1-x^2}} + \sqrt{\sqrt{1-x^2}}$$

$$1-x^2 \geq 0 \quad x^2 \leq 1 \Rightarrow |x| \leq 1 \Rightarrow -1 \leq x \leq 1 \quad (I)$$

$$(y) \quad 1-\sqrt{1-x^2} \geq 0 \quad \sqrt{1-x^2} \leq 1 \quad 1-x^2 \leq 1 \quad x^2 \geq 0$$

$$D(f+g) \circ f = [-1, 1] \checkmark$$

(9) سوال

$$ج) f\left(\frac{x+1}{x-1}\right) = x+1 \quad \frac{x+1}{x-1} = t \quad x = -\frac{-t-1}{t-1} = \frac{t+1}{t-1}$$

$$f(t) = f\left(\frac{t+1}{t-1}\right) + 1 = \frac{1t + 1 + 1}{t-1} = \frac{t+2}{t-1} \Rightarrow f(x) = \frac{x+1}{x-1} \checkmark$$

(y)

$$ب) f\left(x + \frac{1}{x}\right) = x^p + \frac{1}{x^p} = \left(x + \frac{1}{x}\right) \left(x^{p-1} + \frac{1}{x^{p-1}}\right) = \left(x + \frac{1}{x}\right) \left(x + \frac{1}{x}\right)^{p-1}$$

$$x + \frac{1}{x}^p = x^p + \frac{1}{x^p} + p \Rightarrow f(x) = (x) \left(x + \frac{1}{x}\right)^{p-1} \checkmark$$

(10) سوال

$$g(\sqrt{p}) = 0$$

$$g(1) = 0$$

$$f(x) = x\sqrt{x}$$

$$g \circ f = 0 \quad g(f(x)) = 0$$

$$\begin{cases} g(1) = 0 \\ g(\sqrt{p}) = 0 \\ g(f(x)) = 0 \end{cases}$$

$$\Rightarrow f(x) = 1 \Rightarrow x\sqrt{x} = 1 \Rightarrow x_1 = 1$$

$$f(x) = \sqrt{p} \Rightarrow x\sqrt{x} = \sqrt{p} \Rightarrow x_2 = p \checkmark$$

$$|x_2 - x_1| = |p - 1| = 1 \checkmark$$

(y)