

$$f(x) = \begin{cases} \cot \frac{\pi x}{2} & ; x \leq 1 \\ \sqrt{x^2 + 1} & ; x > 1 \end{cases}$$

$$\rightarrow f \circ f \left(\frac{1}{2} \right) = ? \quad (1)$$

$$\textcircled{I} \rightarrow f \left(\frac{1}{2} \right) \quad \frac{1}{2} \leq 1 \rightarrow f \left(\frac{1}{2} \right) = \cot \frac{\pi}{4} = \cot \frac{\pi}{4} = \cot 45^\circ = \frac{\cos 45^\circ}{\sin 45^\circ} = \frac{\frac{\sqrt{2}}{2}}{\frac{1}{2}} \quad (2)$$

$$\textcircled{II} \rightarrow f \circ f \left(\frac{1}{2} \right) = f \left(f \left(\frac{1}{2} \right) \right) = f(\sqrt{2}) \quad \sqrt{2} > 1 \rightarrow f(\sqrt{2}) = \sqrt{(\sqrt{2})^2 + 1} = \sqrt{3} = f \circ f \left(\frac{1}{2} \right)$$

$$\text{ان) } f \left(\frac{1}{x} \right) = \sqrt{\frac{x^2 - 1}{x^2}} \quad , \quad g(x) = 2 \cos^2 x \rightarrow f \circ g \left(\frac{\pi}{4} \right) = ? \quad (3)$$

$$f \circ g \left(\frac{\pi}{4} \right) = f \left(g \left(\frac{\pi}{4} \right) \right) \rightarrow \begin{cases} \textcircled{I} g \left(\frac{\pi}{4} \right) = g(45^\circ) = 2(\cos^2 45^\circ) = 2 \left(\frac{1}{2} \right) = 1 \\ \textcircled{II} f \left(\frac{1}{2} \right) = \sqrt{\frac{2^2 - 1}{2^2}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2} = f \circ g \left(\frac{\pi}{4} \right) \end{cases}$$

$$\text{ب) } f(x) = \lfloor x \rfloor \quad , \quad g(x) = \frac{x}{x-1} \rightarrow f \circ g(\sqrt{2}) = ?$$

$$f \circ g(\sqrt{2}) = f \left(g(\sqrt{2}) \right) \rightarrow \begin{cases} \textcircled{I} g(\sqrt{2}) = \frac{\sqrt{2}}{\sqrt{2}-1} \xrightarrow{\text{مضروب}} \frac{\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{2+\sqrt{2}}{2-1} = 2+\sqrt{2} \\ \textcircled{II} f(2+\sqrt{2}) = \lfloor 2+\sqrt{2} \rfloor = 3 = f \circ g(\sqrt{2}) \end{cases}$$

$$f(x) = \sin x \quad , \quad g(x) = x\sqrt{1-x^2} \rightarrow g \circ f \left(\frac{\pi}{2} \right) = ?$$

$$g \circ f \left(\frac{\pi}{2} \right) = g \left(f \left(\frac{\pi}{2} \right) \right) = \begin{cases} \textcircled{I} f \left(\frac{\pi}{2} \right) = f(90^\circ) = \sin 90^\circ = 1 \\ \textcircled{II} g(1) = 1 \cdot \sqrt{1-1^2} = 1 \cdot 0 = 0 = g \circ f \left(\frac{\pi}{2} \right) \end{cases}$$

$$g(x) = \{(x, y) | (x, y) \in \{(1, 2), (2, 1), (3, 1), (4, 1)\}\} \quad , \quad f(x) = \{(x, y) | (x, y) \in \{(1, 2), (2, 1), (3, 1), (4, 1)\}\} \quad (4)$$

$$\text{الف) } f \circ g(x) = \{(1, 2), (2, 1), (3, 1), (4, 1)\}$$

$$\text{ب) } g \circ f(x) = \{(1, 2), (2, 1), (3, 1), (4, 1)\} = \emptyset$$

$$\text{ج) } f \circ f(x) = \{(1, 2), (2, 1), (3, 1), (4, 1)\} = \emptyset$$

$$\text{د) } g \circ g(x) = \{(1, 2), (2, 1), (3, 1), (4, 1)\} = \{(1, 2), (2, 1)\}$$

$$f = \{(1,1), (2,2), (3,3), (4,4)\}, g = \{(1,2), (2,1), (3,3), (4,4)\}$$

if $(1,2) \in f \circ g, (1,1) \in g \circ f$

$$\text{نحوه } (1,2) \in f \circ g \rightarrow \text{if } m=1 \rightarrow g(1)=2 \rightarrow f(2)=2 \rightarrow \square = 2 \rightarrow m=1 \rightarrow \square = 2$$

$$\text{نحوه } (1,1) \in g \circ f \rightarrow \text{if } n=1 \rightarrow f(1)=1 \rightarrow g(1)=2 \rightarrow \square = 2 \rightarrow n=1 \rightarrow \square = 1$$

$$\text{نحوه } (a,b) = (1,2)$$

$$f(f(m)) = f(m+1), g(g(n)) = g(n-1) \rightarrow g \circ f(-1) = ?$$

$$\text{① } f(m) = am+b \Rightarrow f(f(m)) = f(am+b) = a(am+b) + b = a^2m + ab + b$$

$$\text{نحوه } f(f(m)) = f(m+1) \rightarrow a^2m + ab + b = f(m+1) \rightarrow a^2 = 1, ab + b = b(a+1) = 1$$

$$\Rightarrow \text{if } \begin{cases} a=1 \rightarrow b(1+1)=1 \Rightarrow b=1/2 \rightarrow f(m) = m + 1/2 \\ a=-1 \rightarrow b(-1+1)=1 \Rightarrow b=1/0 \rightarrow f(m) = -m - 1 \end{cases}$$

$$\rightarrow f(-1) = \begin{cases} \text{مورد اول} \rightarrow (-1) + 1/2 = -1/2 \\ \text{مورد دوم} \rightarrow -(-1) - 1 = 0 \end{cases} \rightarrow f(-1) = 0$$

$$\rightarrow g(-1) = 2, f(2) = 2 \Rightarrow g(-1) = f(2) = 2 \Rightarrow \boxed{-1} = g \circ f(-1)$$

$$f(m) = \sqrt{m+1}, g(m) = \frac{1}{m^2-1} \rightarrow g \circ f = ?$$

$$f(m) = \sqrt{m+1} \Rightarrow m+1 \geq 0 \rightarrow \begin{cases} m \geq -1 \rightarrow \sqrt{m+1} \\ m < -1 \rightarrow \text{undefined} \end{cases} \rightarrow D_{f(m)} = [-1, \infty)$$

$$g(m) = \frac{1}{m^2-1} \Rightarrow m^2-1 \neq 0 \Rightarrow m \neq \pm 1 \rightarrow D_{g(m)} = \mathbb{R} - \{0, \pm 1\}$$

$$\text{for } m \in D_{f(m)} \cap D_{g(m)} \rightarrow \begin{cases} m < -1 \rightarrow -m = |m| \rightarrow f(m) = \sqrt{-m+1} = 0 \\ m \geq -1 \rightarrow m = |m| \rightarrow f(m) = \sqrt{m+1} = \sqrt{m+1} \end{cases}$$

$$\text{نحوه } D_{g(m)} \cap D_{f(m)} \rightarrow \sqrt{m+1} \neq 0 \rightarrow m \neq -1 \rightarrow \sqrt{m+1} \neq 1 \rightarrow m \neq 0$$

$$\boxed{D_{f \circ g} = \{m \in \mathbb{R} \mid m \neq -1, 0\}}$$

$$\rightarrow D_{f \circ g} = \{m \in \mathbb{R} \mid m \neq -1, 0\}$$

$$f(x) = \sqrt{1-x^2}$$

$$g(x) = \sqrt{x} \rightarrow D(f+g) = ?$$

(1)

$$\rightarrow f(x) = \sqrt{1-x^2} \rightarrow 1-x^2 \geq 0 \Rightarrow x^2 \leq 1 \Rightarrow -1 \leq x \leq 1 \rightarrow D_f = [-1, 1]$$

$$\rightarrow g(x) = \sqrt{x} \rightarrow x \geq 0 \Rightarrow D_g = [0, +\infty) \Rightarrow D(f+g) = D_f \cap D_g = [0, 1]$$

$$\rightarrow (f+g)(x) \rightarrow 0 \leq \sqrt{1-x^2} \leq 1 \Rightarrow D(f+g) = [0, 1]$$

$$الف) f\left(\frac{x+1}{x-2}\right) = x-2 \rightarrow \frac{x+1}{x-2} = t \Rightarrow x = \frac{t^2+1}{t-2}$$

(2)

$$\rightarrow f(t) = \frac{-t^2+1}{t-2} \cdot \frac{-t^2+1}{t-2} \rightarrow f(t) = \frac{t^4+1}{t-2} \rightarrow f(t) = \frac{x^2+1}{x-2}$$

$$\Rightarrow \frac{t^2+1}{t-2} + \frac{t^2+1}{t-2} = \frac{t^4+1}{t-2} \Rightarrow f(x) = \frac{x^4+1}{x-2}$$

$$ب) f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} \rightarrow \left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 \Rightarrow x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$$

$$\Rightarrow x + \frac{1}{x} = t \rightarrow f(t) = t^2 - 2 \Rightarrow f(x) = x^2 - 2$$

$$g(x) = (x-1)(x-\sqrt{x}) = x^2 + (-1-\sqrt{x})x + \sqrt{x} \Rightarrow f(x) = x\sqrt{x} = \sqrt{x^3}$$

(3)

نوار
لا، Com log نوار $\rightarrow g(f(x)) = 0 \rightarrow f(x) = 1 \vee f(x) = \sqrt{x}$

نقطة
مفتوح

$$\left. \begin{array}{l} \text{I) } \sqrt{x^2} = 1 \Rightarrow x = 1 \\ \text{II) } \sqrt{x^2} = \sqrt{x} \Rightarrow x = 2 \end{array} \right\} \rightarrow x_2 - x_1 = 2 - 1 = 1$$