تاریخ A   کلاس ۹   ۱۸٫۵   علیرضا ...

۱)

$f(x)=\begin{cases}\cot\frac{x\pi}{4} & ; x\le 1\\ \sqrt{x^2+1} & ; x>1\end{cases}$

$\frac{2}{3}<1 \rightarrow f(\frac{2}{3})=\cot\frac{2\pi}{12}=\cot\frac{\pi}{6}=\sqrt{3}$

$\sqrt{3}>1 \rightarrow f(\sqrt{3})=\sqrt{(\sqrt{3})^2+1}=\sqrt{3+1}=2$ ✓

$fof(\frac{2}{3})=? = f(\sqrt{3})=2$

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۲)

$f(\frac{1}{x})=\sqrt{\frac{2x-1}{x^2}} \quad f(\frac{1}{2})=\sqrt{\frac{2\times2-1}{2^2}}=\frac{\sqrt{3}}{2}$

$g(x)=2\cos^2 x \rightarrow g(\frac{\pi}{3})=2\cos^2\frac{\pi}{3}=\frac{1}{2}$

$fog(\frac{\pi}{3})=f(\frac{1}{2})=\frac{\sqrt{3}}{2}$ ✓

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$f(x)=[x] \quad f(-3-\sqrt{3})=[-3-\sqrt{3}]=-4$

$g(x)=\frac{2x}{1-x} \quad g(\sqrt{3})=\frac{2\sqrt{3}}{1-\sqrt{3}}\times\frac{1+\sqrt{3}}{1+\sqrt{3}}=-3-\sqrt{3}$

$fog(\sqrt{3})=f(-3-\sqrt{3})=-4$ ✓

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۳)

$f(x)=\sin x \rightarrow f(\frac{\pi}{4})=\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

$g(x)=x\sqrt{1-x^2} \quad g(\frac{\sqrt{2}}{2})=\frac{\sqrt{2}}{2}\sqrt{1-(\frac{\sqrt{2}}{2})^2}=\frac{\sqrt{\frac{1}{2}}}{\sqrt{2}}=\frac{1}{2}$

$gof(\frac{\pi}{4})=? = g(\frac{\sqrt{2}}{2})=\frac{1}{2}$ ✓

(۲)

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۴)

$f(x)=\{(4,5)(6,5)(8,12)(10,12)\} \quad g(x)=\{(4,6)(2,4)(6,8)(8,10)\}$

$fog=\{(4,5)(2,5)(6,12)(8,12)\}$ ✓ $\quad gof=\varnothing$ ✓

(۲)

$fof=\varnothing$ ✓ $\quad gog=\{(2,6)(4,8)(6,10)\}$ ✓

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۵)

$f=\{(2,1)(4,4)(4,5)(1,7)\} \quad g=\{(1,2)(4,1)(a,4)(b,1)\}$

$(4,2)\in fog$

$(4,1)\in gof$

$\rightarrow fog(4)=2 \rightarrow f(g(4))=2 \rightarrow g(4)=3 \rightarrow a=3 \rightarrow a=4$

$(3,5)$

$gof(4)=1 \rightarrow g(f(4))=1 \rightarrow \begin{cases} f(4)=b \rightarrow b=5 \\ f(4)=3 \quad X\end{cases}$

$(a,b)\rightarrow(4,5)$

(۱)

۷)

$f(f(x)) = 4x + 3 \xrightarrow{f(x)=ax+b} a(ax+b)+b = 4x+3$

$a^2x + ab + b = 4x + 3$ ⟨ $a^2 = 4 \to a = \pm 2$; $b(a+1) = 3$ ⟨ $3b = 3 \to b = 1$; $-b = 3 \to b = -3$

$\Rightarrow \begin{cases} a = 2 & b = 1 \to f(x) = 2x+1 \\ a = -2 & b = -3 \to f(x) = -2x-3 \end{cases}$ ✓

$\hookrightarrow f(-1) = \begin{cases} -1 \\ -1 \end{cases} \to f(-1) = -1$

$g(2x+4) = 3x - 2$, $\{x = -3$

$g(-1) = 3(-3) - 2 = -11$ ... $= -8$

$g(f(-1)) = ? = g(-1) = -8$ ✓

(۲)

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۸)

$f(x) = \sqrt{x+|x|}$

$x + |x| \geq 0 \to D = \mathbb{R}$

$g(x) = \dfrac{1}{x^2 - 4x} \to D_g = \mathbb{R} - \{0, 4\}$

$x^2 - 4x \neq 0$

$x \neq 4, 0$

$\sqrt{x+|x|} \neq 0, 4$ ⟨ $x + |x| \neq 0 \to x > 0$; $x + |x| \neq 16 \to x \neq 8$

$D_{gof} = ?$ چه ایکس‌هایی تعریف شدن در f و g که خروجی آن‌ها در دامنه g تعریف شده $= (0, +\infty) - \{8\}$ ✓

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۸)

$f(x) = \sqrt{1-x^2}$ $1 - x^2 \geq 0 \to -1 \leq x \leq 1$

$g(x) = \sqrt{x}$ $\to x \geq 0$

اشتراک ⟩ = دامنه $0 \leq x \leq 1 \to 0 \leq \sqrt{1-x^2} \leq 1$

$1 - x^2 \leq 1$

$x^2 \geq 0 \to \mathbb{R}$ ✓

$D_{(f+g)of} = \{x \in D_f \mid f(x) \in D_{f+g}\}$

$D(f+g)(f(x)) = [0, 1]$

$\hookrightarrow D_{(f+g)of} = D_f = [-1, 1]$

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۹)

$f\left(\dfrac{3x+1}{x-2}\right) = 4x + 5 \to f(t) = \dfrac{8t+4}{t-3} + 5$

$t = \dfrac{3x+1}{x-2}$

$\to x = +\dfrac{+2t+1}{t-3}$

$\to f(x) = \dfrac{8x+4}{x-3} + 5$

$f(x) = \dfrac{13x - 11}{x - 3}$ ✓

$f\left(x + \frac{1}{x}\right) = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) - 3\left(x + \frac{1}{x}\right)$

$\left(x + \frac{1}{x}\right)^3$

$f\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$

$f(x) = x^3 - 3x$ ✓

(۲)

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۱۰)

برخورد با محور x : $|\frac{2\sqrt{2}}{0}|$ و $|\frac{1}{0}|$

$f(x) = x\sqrt{x}$

$g(f(x)) = 0 \to f(x) \begin{cases} = 1 \to x\sqrt{x} = 1 \to x = 1 \\ = 2\sqrt{2} \to x\sqrt{x} = 2\sqrt{2} \to x = 2 \end{cases}$ ✓

برخورد gof با محور x $\to |\frac{2}{0}|, |\frac{1}{0}| \to$ اختلاف طول = $2 - 1 = 1$ ✓

(۲)