نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

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$fof(\frac{2}{3}) \rightsquigarrow f(\frac{2}{3}) = \cot \frac{\pi 2}{12}$

$f(\frac{2}{3}) = \sqrt{3} \quad \rightsquigarrow \quad f(\sqrt{3}) = \sqrt{\sqrt{3}^2 + 1} = 2$

$\boxed{fof(\frac{2}{3}) = 2}$

۲ الف)

$fog(\frac{\pi}{3}) \rightsquigarrow g(\frac{\pi}{3}) = 2\sin^2 \frac{1}{2} = \frac{1}{2} \rightsquigarrow f(\frac{1}{2}) = \frac{\sqrt{2\sin^2 2 - 1}}{2} = \frac{\sqrt{3}}{2}$

۲ ب)

$fog(\sqrt{2}) \rightsquigarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \subset -3, -\infty \rightsquigarrow f(-3, -\infty) = [-3, -\infty] = -4$

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$gof(\frac{\pi}{4}) \rightsquigarrow f(\frac{\pi}{4}) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2} \times \sqrt{1 - \frac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{1}{2}$

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$fog(x) = \{(5,4), (2,5), (4,12), (7,12)\}$

$gof(x) = \emptyset$

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$gog(x) = \{(8,4), (4,2), (10,4)\}$

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$(a,2), (3,2) \rightsquigarrow (4,2) \in fog(x) \rightsquigarrow a = 4$

$(4,5), (b,1) \rightsquigarrow (4,1) \in gof(x) \rightsquigarrow b = 5$

$(a,b) = (4,5)$

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$fof = a(ax+b)+b \Rightarrow a^2x + ab + b = 4x + 3$ $\quad a^2 = 4 \quad a = \pm 2$

$ab + b = 3 \quad b = -3, 1$

$gof(-1) = ?$

$f(x) = 2x+1$ یا $f(x) = -2x-3 \Rightarrow f(-1) = -1$

$g(2x+3) = 3x-2$

$2x+3 = -1 \Rightarrow x = -2$

$3(-2) - 2 = -8$

$gof(-1) = -8$

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$D_f : \mathbb{R} \qquad \sqrt{x+|x|} \qquad x+|x| \geq 0 \quad \mathbb{R}$

$D_g : \mathbb{R} - \{0, 4\} \qquad \frac{1}{x^2 - 4x} \qquad x(x-4) \neq 0 \quad x \neq 0, 4$

$D_f \neq 0, 4 \Rightarrow D_f : \mathbb{R}^+ - \{8\}$

$\sqrt{x+|x|} \neq 0$

$\sqrt{x+|x|} \neq 4$

$x \neq$ عدد منفی و 0 , $x \neq 8$

$D_{fog} = \mathbb{R}^+ - \{8\}$ یا $(0, +\infty) - \{8\}$

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$f(x) = \sqrt{1-x^2} \qquad D_f : x \leq 1$

$g(x) = \sqrt{x} \qquad D_g : x \geq 0$

$\cup \; [0, 1]$

$D_{fog} = [0, 1] \Rightarrow D_{(fog)of} = [-1, 1]$

$x \in [-1, 1] \Rightarrow 0 \leq f(x) \leq 1$

الف)

$f\left(\frac{3x+1}{x-2}\right) \Rightarrow \frac{3x+1}{x-2} = t \Rightarrow \times \; x-2$

$t \times x - 2 = 3x+1$

$tx - 3x = 2t + 1$

$x = \frac{2t+1}{t-3}$

$f(t) = 4x + 5$

$f(t) = 4\left(\frac{2t+1}{t-3}\right) + 5 = \frac{13t - 11}{t-3}$ یا $\frac{13x-11}{x-3}$

$f\left(x + \frac{1}{x}\right) = x^3 + \frac{1}{x^3}$, $\quad x + \frac{1}{x} = t$

$t^3 = x^3 + \frac{1}{x^3} + 3x + \frac{3}{x}$

$t^3 = x^3 + \frac{1}{x^3} - 3\left(x + \frac{1}{x}\right)$

$f(x) = x^3 - 3x$

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$gof = 0 \Rightarrow f(x) = 1 \qquad x\sqrt{x} = 1 \qquad x = 1$

$g(1) = 0 \qquad f(x) = 2\sqrt{2} \qquad x\sqrt{x} = 2\sqrt{2} \qquad x = 2$

$g(2\sqrt{2}) = 0$

$x_2 - x_1 = 2 - 1 = 1$