نام و نام خانوادگی .... پاسخنامه تشریحی تکلیف شماره ..۹.. کلاس یازدهم ....

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$$f(x)=\begin{cases}\cot\frac{\pi x}{4} & ; x\le 0\\ \sqrt{x^2+1} & ; x>1\end{cases} \qquad f\circ f\left(\frac{2}{3}\right)$$

$$f\left(\frac{2}{3}\right)=\cot\frac{\pi}{6}=\sqrt{3}\longrightarrow f\circ f\left(\frac{2}{3}\right)=f(\sqrt{3})=\sqrt{4}=2$$

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الف)

$$f\left(\frac{1}{x}\right)=\sqrt{\frac{2x-1}{x^2}} \qquad g(x)=2\cos^2 x$$

$$f\circ g\left(\frac{\pi}{3}\right)?$$

$$g\left(\frac{\pi}{3}\right)=2\times\frac{1}{4}=\frac{1}{2}\Longrightarrow \frac{1}{x}\to x=2$$

$$f\circ g\left(\frac{\pi}{3}\right)=\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$$

ب)

$$f(x)=[x] \qquad g(x)=\frac{x}{1-x}$$

$$f\circ g(\sqrt{2})?$$

$$g(\sqrt{2})=\frac{\sqrt{2}}{1-\sqrt{2}}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}=\frac{2+\sqrt{2}}{-1}=-2-\sqrt{2}$$

$$f\circ g(\sqrt{2})=f(-2-\sqrt{2})=-4$$

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$$f(x)=\sin x \qquad g(x)=x\sqrt{1-x^2} \qquad g\circ f\left(\frac{\pi}{4}\right)?$$

$$f\left(\frac{\pi}{4}\right)=\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}\longrightarrow g\circ f\left(\frac{\pi}{4}\right)=g\left(\frac{\sqrt{2}}{2}\right)=\frac{\sqrt{2}}{2}\sqrt{1-\frac{1}{2}}$$

$$=\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{2}=\frac{1}{2}$$

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$$f(x)=\{(4,5),(6,5),(8,12),(10,12)\} \qquad g(x)=\{(4,6),(2,4),(6,8),(8,10)\}$$

الف) $f\circ g(x)=\{(4,5),(2,5),(6,12),(8,12)\}$

ب) $g\circ f(x)=\{\varnothing\}$

ج) $f\circ f(x)=\{\varnothing\}$

د) $g\circ g(x)=\{(4,8),(2,6),(6,10)\}$

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$$f=\{(2,1),(3,2),(4,5),(1,7)\} \qquad g\{(4,2),(3,4),(a,2),(b,1)\}$$

$(4,2)\in f\circ g$

$f(O)=2\longrightarrow O=3$

$g(4)=3\longrightarrow \boxed{a=4}$

$(4,1)\in g\circ f$

$4\xrightarrow{f}5 \qquad b\xrightarrow{g}1$

$\longrightarrow \boxed{b=5}$

$\longrightarrow (a,b)=(4,5)$

$f(f(x)) = 4x+3 \qquad f = ax+b$ $f(f(x)) = a(ax+b)+b = a^2x+ab+b = 4x+3$ $\to a = \pm 2 <$ $a=2 \to b=1$ ل $\to 2x+1=y \to f(-1)=-1$; $a=-2 \to b=-3$ ل $\to -2x-3=y \to f(-1)=-1$ برابر $g(2x+3) = 3x-2 \to 2x+3=t \to x=\frac{t-3}{2} \to g(t) = 3\,\frac{t-3}{2}-2$ $\to g(t) = \frac{3t-13}{2} \to g(x) = \frac{3x-13}{2} \to gof(-1) = \{g(-1) = -8\}$	۶
$f(x) = \sqrt{x+\lvert x\rvert} \qquad g(x) = \frac{1}{x^2-4x}$ $D_{gof}(x) \xrightarrow{D_f} x+\lvert x\rvert \geq 0 \;,\; D_g = x^2-4x \neq 0 \to x \neq 0, 4$ $\to \sqrt{x+\lvert x\rvert} \neq 0 \to x \notin \mathbb{R}^-, x\neq 0$ $\to \sqrt{x+\lvert x\rvert} \neq 4 \to x \neq 8$ $\Big\} \to D_{gof} = (0 \;\; +\infty) - \{8\}$	۷
$f(x) = \sqrt{1-x^2} \;,\; g(x) = \sqrt{x}$ $f+g = \sqrt{1-x^2} + \sqrt{x}\; of \xrightarrow{D} D_f = [-1 \;\; 1]$ $D_{f+g} \to x \geq 0$ $\qquad D_{f+gof} = [-1, 1]$ $\to \sqrt{1-x^2} \geq 0 \xrightarrow{\text{همیشه}}$	۸
الف) $f\left(\frac{2x+1}{x-2}\right) = 4x+5 \to \frac{2x+1}{x-2} = t \to \frac{2t+1}{t-2} = x \to f(t) = 4\left(\frac{2t+1}{t-2}\right)+5$ $\to f(t) = \frac{8t+4}{t-2}+5 \Rightarrow f(t) = \frac{13t-19}{t-2} \to f(x) = \frac{13x-19}{x-2}$ ب) $f(x+\frac{1}{x}) = x^3+\frac{1}{x^3} \to f(x+\frac{1}{x}) = (x+\frac{1}{x})(x^2-1+\frac{1}{x^2})$ $\to f(x+\frac{1}{x}) = (x+\frac{1}{x})((x+\frac{1}{x})^2-3) \to f(x) = x(x^2-3) = x^3-3x$	۹
g دو نقطه $\to 1, 2\sqrt{2} \qquad f(x) = x\sqrt{x}$ $fog \xrightarrow{\text{دو نقطه}} x\sqrt{x} = 1 \to x = 1$ $x\sqrt{x} = 2\sqrt{2} \to x = 2$ $\Big\} \to$ ۱ ، ۱-۲ ، اضافات	۱۰