

سوال ۱-

$$\text{الف)} 4x^2 - 2x - 4 = 0 \rightarrow x^2 - 2x - 2 = 0 \rightarrow (x-4)(x+1) = 0$$

$$\rightarrow x \begin{cases} \rightarrow \frac{4}{4} = \frac{2}{2} \\ \rightarrow -\frac{4}{4} = -\frac{2}{2} \end{cases}$$

$$\text{ب)} x^2 - 11x - 12 = 0 \Rightarrow x^2 - 11x - 12 = 0 \Rightarrow (x-12)(x+1) = 0 \Rightarrow x \begin{cases} \rightarrow \frac{12}{1} = 12 \\ \rightarrow -\frac{1}{1} = -1 \end{cases}$$

$$\text{ج)} 4x^2 - 4x - 2 = 0 \Rightarrow x^2 - x - \frac{1}{2} = 0 \Rightarrow (x-2)(x+\frac{1}{2}) = 0 \Rightarrow x \begin{cases} \rightarrow \frac{2}{1} = 2 \\ \rightarrow -\frac{1}{2} = -\frac{1}{2} \end{cases}$$

$$\text{د)} 2x^2 - 2x - 12 = 0 \Rightarrow x^2 - x - 6 = 0 \Rightarrow (x-3)(x+2) = 0 \Rightarrow x \begin{cases} \rightarrow \frac{3}{1} = 3 \\ \rightarrow -\frac{2}{1} = -2 \end{cases}$$

$$\text{الف)} x^2 - 2x - 12 = 0 \Rightarrow (x-6)(x+2) = 0 \Rightarrow x < -2$$

سوال ۲:

$$\text{ب)} 2x^2 + 2x + 2 = 0 \Rightarrow a+c=b \rightarrow x < -\frac{1}{2}$$

$$\text{ج)} x^2 + 2x + 2 = 0 \Rightarrow \Delta = b^2 - 4ac \rightarrow 4 - 12 = -8 \rightarrow \Delta < 0 \rightarrow \text{ریشه حقیقی ندارد}$$

$$\text{د)} 2x^2 + 2x - 2 = 0 \quad \Delta = b^2 - 4ac \quad x = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 2 \cdot (-2)}}{4}, \quad x = \frac{-2 - \sqrt{2^2 - 4 \cdot 2 \cdot (-2)}}{4}$$

$$A = \{1, 2, 3\} \quad B = \{2, 3\} \quad AB = \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)\} \quad \text{سوال ۳:}$$

$$A \times B - B^2 \quad B^2 = \{(2, 2), (2, 3), (3, 2), (3, 3)\} \quad A \times B - B^2 = \{(1, 2), (1, 3)\}$$

$$\text{الف)} \Rightarrow m^2 = m + 2 \Rightarrow m^2 - m - 2 = 0 \Rightarrow (m-2)(m+1) = 0 \quad \text{سوال ۴:}$$

$$\Rightarrow m = \begin{cases} \rightarrow 2 \\ \rightarrow -1 \end{cases} \Rightarrow m = -1$$

$$\text{ب)} \Rightarrow m^2 = m + 2 \Rightarrow \begin{cases} \rightarrow 2 \\ \rightarrow -1 \end{cases} \Rightarrow \text{هر دو غیر قابل قبول}$$

سوال ۵:

$$a^2 = 3a - 2 \Rightarrow a^2 - 3a + 2 = 0 \Rightarrow a \begin{cases} \rightarrow 1 \\ \rightarrow 2 \end{cases} \Rightarrow a = 2$$

$$x=1 \rightarrow x^2 - 1 = ax + b \rightarrow a + b = 0 \rightarrow b = -a \quad \text{سوال ۶:}$$

$$x=2 \rightarrow x^2 = ax + b \rightarrow 4 = 2a + b \rightarrow a = 1, b = -1$$

$$ab = -1 \times 1 = -1$$

سوال ۷:

$$\left. \begin{array}{l} 2x-3 \\ 3-2x \end{array} \right\} \begin{array}{l} x \geq k \\ x \geq k \end{array} \Rightarrow 2k-3 = 3-2k \Rightarrow 4k = 6 \Rightarrow k = \frac{3}{2}$$

سوال ۸: تابع نیست

الف) $y^3 - 4y = x^2 - 4x + 3 \xrightarrow{x=-1} y^3(y-4) = 0 \Rightarrow \begin{cases} y=0 \\ y=4 \end{cases}$

ب) $|x| + |y| = 3 \xrightarrow{x=1} y = \pm 2 \rightarrow$ تابع نیست

ج) $|y| = \sqrt{1 - \cos x} \xrightarrow{x=180^\circ} y = \pm 1 \rightarrow$ تابع نیست

د) $-y^2 = x^2 + 2x + 1 \rightarrow (x+1)^2 \rightarrow (-1, 0)$ تابع هست

سوال ۹:

الف) $\sqrt[3]{y} + x^2 = 2x - 2 \rightarrow \sqrt[3]{y} = -x^2 + 2x - 2$

$\Rightarrow \left. \begin{array}{l} \sqrt[3]{y_1} = x_1^2 + 2x_1 - 2 \\ \sqrt[3]{y_2} = -x_2^2 + 2x_2 - 2 \end{array} \right\} \rightarrow x_1 = x_2 \rightarrow \sqrt[3]{y_1} = \sqrt[3]{y_2}$

یا $y_1 = y_2$ تابع هست

ب) $y^2 + 2y = 2x \Rightarrow \begin{array}{l} y_1^2 + 2y_1 = 2x_1 \\ y_2^2 + 2y_2 = 2x_2 \end{array} \xrightarrow{x_1=x_2} y_1^2 + 2y_1 = y_2^2 + 2y_2$

$y_1^2 - y_2^2 + 2y_1 - 2y_2 = 0 \rightarrow (y_1 - y_2)(y_1 + y_2 + 2) + 2(y_1 - y_2) = 0$

$\rightarrow \underbrace{(y_1 - y_2)}_{y_1=y_2} \underbrace{(y_1 + y_2 + 2 + 2)}_{\text{معمولاً}} = 0 \rightarrow y_1 = y_2$ تابع هست

سوال ۱۰:

الف) $x \sin y = y \sin x \xrightarrow{x=0} 0 \times \sin y = y \times 0 \Rightarrow R=y$ تابع نیست

ب) $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2 \xrightarrow{\text{تکثیر}} \frac{x}{y} + 2 + \frac{y}{x} = 4 \Rightarrow \frac{x^2 + y^2}{xy} - 2 = 0$

$x^2 + y^2 - 2xy = 0 \rightarrow (x-y)^2 = 0 \Rightarrow x = y$

