

الف) $4n^2 - 5n - 9 = 0 \Rightarrow n^2 - 5n - 9 = 0 \Rightarrow (n-9)(n+4) = 0 \Rightarrow n = \frac{9}{4} \text{ و } \frac{4}{9} \Rightarrow n = \frac{9}{4} \text{ و } \frac{4}{9}$

ب) $3n^2 - 12n - 15 = 0 \Rightarrow n^2 - 4n - 5 = 0 \Rightarrow (n-5)(n+1) = 0 \Rightarrow n = \frac{4}{3} \text{ و } \frac{1}{3} \Rightarrow n = \frac{4}{3} \text{ و } \frac{1}{3}$

ج) $5n^2 - 12n - 15 = 0 \Rightarrow n^2 - 4n - 3 = 0 \Rightarrow (n-4)(n+3) = 0 \Rightarrow n = \frac{4}{5} \text{ و } \frac{3}{5} \Rightarrow n = \frac{4}{5} \text{ و } \frac{3}{5}$

د) $2n^2 - 5n - 12 = 0 \Rightarrow n^2 - 5n - 12 = 0 \Rightarrow (n-1)(n+12) = 0 \Rightarrow n = \frac{1}{2} \text{ و } \frac{12}{2} \Rightarrow n = \frac{1}{2} \text{ و } 6$

الف) $n^2 - 5n - 12 = 0 \Rightarrow (n-5)(n+12) = 0 \Rightarrow n = 5 \text{ و } -12$

ب) $2n^2 + 5n + 3 = 0 \Rightarrow n^2 + 5n + 9 = 0 \Rightarrow (n+2)(n+3) = 0 \Rightarrow n = \frac{-2}{2} \text{ و } -1 \Rightarrow n = -1 \text{ و } -3$

ج) $n^2 + 2n + 3 = 0 \quad \Delta < 0 \quad \text{ریشه ندارد}$

د) $3n^2 + 2n - 2 = 0 \quad \Delta = 4 - 4 \times 3 \times (-2) = 4 + 24 = 28 \Rightarrow n = \frac{-2 \pm \sqrt{28}}{6}$

$A = \{1, 2, 3\}$ و $B = \{2, 3\}$

$A \times B = \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)\}$

$B^2 = \{(2, 2), (2, 3), (3, 2), (3, 3)\}$

$A \times B - B^2 = \{(1, 2), (1, 3)\}$

$m^2 = m + 2 \Rightarrow m^2 - m - 2 = 0 \Rightarrow (m-2)(m+1) = 0 \Rightarrow m = 2 \text{ و } -1$

الف)

$m = -1$

$m^2 = m + 2 \Rightarrow m = 2 \text{ و } -1 \Rightarrow (-1, 2), (-1, 3)$

ب)

m متغیر ندارد

$f(n) = \{(1, a^2), (1, 3a-2), (a^2, 3a+2)\}$

$a^2 = 3a - 2 \Rightarrow a^2 - 3a + 2 = 0 \Rightarrow (a-2)(a-1) = 0 \Rightarrow a = 2 \text{ و } 1$

$a = 2$

$$f(n) = \begin{cases} n^2 - 1 & n < 1 \\ an + b & 1 \leq n \leq 2 \\ n^3 & n > 2 \end{cases}$$

$$\left. \begin{aligned} n=1 &\Rightarrow 0 = a+b \\ n=2 &\Rightarrow 1 = 2a+b \end{aligned} \right\} \Rightarrow \begin{aligned} a &= 1 \\ b &= -1 \end{aligned}$$

$$a+b = -4 \varepsilon$$

6

$$f(x) = \begin{cases} 2x-3 & x \geq K \\ 4-3x & x < K \end{cases}$$

$$\begin{aligned} x=K &\Rightarrow 2K-3 = 4-3K \\ \Rightarrow 5K &= 7 \Rightarrow K = \frac{7}{5} \end{aligned}$$

7

الف) $y^3 - 4y = x^2 - 4x + 3$ $x=1 \Rightarrow y^3 - 4y = 0$ $y(y^2 - 4) = 0$ $y = 0, \pm\sqrt{4}$ تابع نیست

ب) $|x| + |y| = 3$ $x=0 \Rightarrow y = \pm 3$ تابع نیست

ج) $|y| = \sqrt{1 - \cos x}$ $1 - \cos x \geq 0$ $1 \geq \cos x$ $x=0 \Rightarrow y = \pm 1$ تابع نیست

8

د) $y^2 + x^2 + 2x + 1 = 0 \Rightarrow y^2 + (x+1)^2 = 0$ $x=-1, y=0$ تابع هست
مجموعه فقط یک نقطه است پس هر دو صفر بوده

الف) $\sqrt[3]{y} + x^2 = 2x - 2$ $x_1 = x_2 \Rightarrow \sqrt[3]{y_1} = \sqrt[3]{y_2} \Rightarrow y_1 = y_2$ تابع است
 $\sqrt[3]{y_1} = -x_1^2 + 2x_1 - 2$
 $\sqrt[3]{y_2} = -x_2^2 + 2x_2 - 2$

ب) $y^3 + y^2 \cos x$ $x_1 = x_2 \Rightarrow y_1^3 + y_1^2 = y_2^3 + y_2^2 \Rightarrow (y_1^3 - y_2^3) + (y_1^2 - y_2^2) = 0$ تابع هست
 $(y_1 - y_2)(y_1^2 + y_1 y_2 + y_2^2) + (y_1 - y_2)(y_1 + y_2) = 0 \Rightarrow (y_1 - y_2)(y_1^2 + y_1 y_2 + y_2^2 + y_1 + y_2) = 0$
 $\Rightarrow y_1 = y_2$ $y_1^2 - 4(y_1^2 + 2) < 0 \Rightarrow -3y_1^2 - 8 < 0$

9

الف) $x \sin y = y \sin x$ $x=0 \Rightarrow 0 = y \sin 0 \Rightarrow y \in \mathbb{R}$ تابع نیست

10

ب) $\frac{\sqrt{x}}{a} + \frac{\sqrt{y}}{\frac{1}{a}} = 2 \Rightarrow a + \frac{1}{a} = 2 \Rightarrow a^2 - 2a + 1 = 0 \Rightarrow (a-1)^2 = 0$ $a=1$ تابع هست
 $\frac{\sqrt{x}}{a} = 1 \Rightarrow \frac{x}{y} = 1 \Rightarrow x = y$ برای همگی مخالف صفر