

نام و نام خانوادگی علی حبیبیان پاسخنامه تشریحی تکلیف شماره ۵... کلاس بی بی

الف) $6x^2 - 5x - 6 = 0 \rightarrow x^2 - 5x - 3 = 0 \rightarrow (x-9)(x+4) = 0 \Rightarrow x = \begin{cases} -\frac{4}{9} = -\frac{2}{9} \\ 9 = \frac{2}{9} \end{cases}$

ب) $3x^2 - 8x - 3 = 0 \rightarrow x^2 - 8x - 9 = 0 \rightarrow (x-9)(x+1) = 0 \Rightarrow x = \begin{cases} -\frac{1}{9} = -\frac{1}{9} \\ 9 = \frac{1}{9} \end{cases}$

ج) $4x^2 - 4x - 3 = 0 \rightarrow x^2 - 4x - 12 = 0 \rightarrow (x-6)(x+2) = 0 \Rightarrow x = \begin{cases} -\frac{2}{6} = -\frac{1}{3} \\ 6 = \frac{2}{6} \end{cases}$

د) $2x^2 - 5x - 12 = 0 \rightarrow x^2 - 5x - 12 = 0 \rightarrow (x-8)(x+3) = 0 \Rightarrow x = \begin{cases} \frac{3}{8} = \frac{3}{8} \\ -\frac{3}{8} = -\frac{3}{8} \end{cases}$

الف) $x^2 - 5x - 14 = 0 \rightarrow (x-7)(x+2) = 0 \Rightarrow x = \begin{cases} -2 \\ 7 \end{cases}$

ب) $2x^2 + 5x + 3 = 0 \rightarrow x^2 + 5x + 6 = 0 \rightarrow (x+2)(x+3) = 0 \Rightarrow x = \begin{cases} -3 \\ -2 \end{cases}$

ج) $x^2 + 2x + 3 = 0$ $\Delta = b^2 - 4ac = 4 - 12 = -8$ ریشه حقیقی ندارد!

د) $3x^2 + 3x - 2 = 0$ $\Delta = b^2 - 4ac = 9 + 24 = 33 \Rightarrow x = \begin{cases} \frac{-3 + \sqrt{33}}{6} \\ \frac{-3 - \sqrt{33}}{6} \end{cases}$

$A = \{1, 2, 3\}$ $B = \{2, 3\}$

$A \times B - B^2 = \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)\} - \{(2, 2), (2, 3), (3, 2), (3, 3)\}$

$= \{(1, 2), (1, 3)\}$

الف) $f(x) = \{(3, m^2), (2, 1), (-3, m), (-2, m), (3, m+2), (m, 4)\} \rightarrow m^2 = m+2 \Rightarrow m = \begin{cases} 2 \\ -1 \end{cases} \rightarrow m = -1$

ب) $g(x) = \{(3, m^2), (2, 1), (m, 4), (3, m+2), (-2, m), (-1, 3)\} \rightarrow m^2 = m+2 \Rightarrow m = \begin{cases} 2 \\ -1 \end{cases} \rightarrow m = -1$ غیر ممکن!

$f(x) = \{(1, a^2), (1, 3a-2), (a^2, 3a+2)\} \rightarrow a^2 = 3a-2 \rightarrow a^2 - 3a + 2 = 0 \rightarrow (a-2)(a-1) = 0 \Rightarrow a = \begin{cases} 1 \\ 2 \end{cases}$

اگر $a=1 \rightarrow f(x) = \{(1, 1), (1, 1), (1, 5)\}$ ✗

اگر $a=2 \rightarrow f(x) = \{(1, 1), (1, 4), (4, 8)\}$ ✓ فقط $a=2$ تابع می شود $\Rightarrow$

$f(x) = \begin{cases} x^2 - 1 & x \leq 1 \\ ax + b & 1 \leq x \leq 2 \\ x^2 & x \geq 2 \end{cases}$ if $x=1 \Rightarrow x^2 - 1 = ax + b \rightarrow 1 - 1 = a + b \Rightarrow a + b = 0$ if $x=2 \Rightarrow ax + b = x^2 \rightarrow 2a + b = 4 \Rightarrow (a+b) - (a+b) = 4 - 0 \Rightarrow a = 4$ if $a=4 \Rightarrow b = (a+b) - 4 = 0 - 4 = -4 \Rightarrow a \times b = -4 \times 4 = -16$	۶
$f(x) = \begin{cases} 2x - 3 & x \geq k \\ 4 - 3x & x \leq k \end{cases}$ if $x=k \Rightarrow 2k - 3 = 4 - 3k \rightarrow 5k = 7 \Rightarrow k = \frac{7}{5}$	۷
الف) $y^2 - 6y = x^2 - 4x + 3$ $\xrightarrow{x_1=x_2} y_1^2 - 6y_1 = y_2^2 - 6y_2 \rightarrow y_1^2 - y_2^2 = 6y_1 - 6y_2 \rightarrow (y_1 - y_2)(y_1^2 + y_1y_2 + y_2^2) = 6(y_1 - y_2)$ $(y_1 + y_2 + y_2^2) = 6(y_1 - y_2)$ تابع نیست ب) $x + y = 3$ $\xrightarrow{x=0} y = 3 \Rightarrow y = \pm 3$ تابع نیست ج) $y = \sqrt{1 - \cos x}$ $\xrightarrow{x=\frac{\pi}{2}} y = 1 \Rightarrow y = \pm 1$ تابع نیست د) $y^2 + x^2 + 2x + 1 = 0$ $y^2 = -(x^2 + 2x + 1)$ $x^2 + 2x + 1 \geq 0 \Rightarrow$ تابع است	۸
الف) $\sqrt[3]{y} + x^2 = 2x - 2$ $x_1 = x_2 \rightarrow \sqrt[3]{y_1} = \sqrt[3]{y_2} \rightarrow \sqrt[3]{y_1^3} = \sqrt[3]{y_2^3} \Rightarrow y_1 = y_2$ تابع است ب) $y^2 + 2y = 5x$ $x_1 = x_2 \rightarrow y_1^2 + 2y_1 = y_2^2 + 2y_2 \rightarrow y_1^2 - y_2^2 = 2y_2 - 2y_1 \rightarrow (y_1 - y_2)(y_1^2 + y_1y_2 + y_2^2) = 2(y_2 - y_1)$ $(y_1^2 + y_1y_2 + y_2^2) = -2$ $y_1^2 + y_1y_2 + y_2^2 = -2 \rightarrow$ غیر ممکن $y_1 = y_2 \rightarrow$ تابع نیست	۹
الف) $x \sin y = y \sin x \rightarrow \frac{\sin y}{y} = \frac{\sin x}{x} \xrightarrow{x_1=x_2} \frac{\sin y_1}{y_1} = \frac{\sin y_2}{y_2} \xrightarrow{\text{if } y_1 = \frac{\pi}{2}} \frac{1}{\frac{\pi}{2}} = \frac{-1}{-\frac{\pi}{2}} y_1 = -y_2$ تابع نیست ب) $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2$ $\frac{x}{y} + \frac{y}{x} + 2\sqrt{\frac{xy}{xy}} = 4 \rightarrow \frac{x}{y} + \frac{y}{x} = 2 \rightarrow \frac{x^2 + y^2}{xy} = 2$ $2xy = x^2 + y^2 \rightarrow x^2 - 2xy + y^2 = 0$ $(x-y)^2 = 0$ $(y-x)^2 = 0$ $\downarrow \quad \downarrow$ $x=y$ تابع است	۱۰