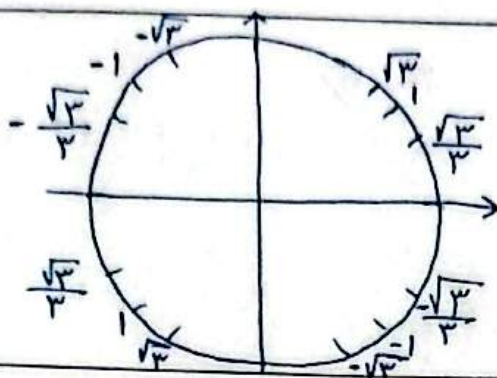




۱) $\sin \frac{\pi}{6} + \cos \pi + 2 \tan \frac{\pi}{6} = \frac{1}{2} + (-1) + 2(-1) = \frac{1}{2} - 2 = -\frac{3}{2} = -1.5$

۲) $-\cos \frac{\pi}{6} + \sin \frac{\pi}{6} - 2 \sin \frac{\pi}{6} = -\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right) - 2\left(\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$

۳) $\cos \frac{\pi}{6} + 2 \cos \frac{\pi}{6} - 2 \tan \frac{\pi}{6} = \frac{\sqrt{3}}{2} + \sqrt{3} - 2(-1) = \frac{\sqrt{3}}{2} + \sqrt{3} + 2 = \frac{3\sqrt{3}}{2} + 2$

۴) $\cos(\sin \pi) + \sin(\pi \cos \frac{\pi}{6}) = \cos(0) + \sin(\pi \times \frac{1}{2}) = \cos(0) + \sin(\frac{\pi}{2}) = 1 + 1 = 2$

$$f(x) = \begin{cases} x^2 + 2x & x \geq a \\ 2x + a + 2 & x < a \end{cases}$$
 $(a)^2 + f(a) = 2(a) + a + 2 \Rightarrow$

$x^2 + 2x = 2x + a + 2 \Rightarrow x^2 + a - 2 = 0$ $\frac{a+b+c=0}{1, -2} \Rightarrow a > 0 \Rightarrow a = 1 \Rightarrow$

$f(1) = (1)^2 + f(1) = 2 \Rightarrow 2(1) + 1 + 2 = 5 \Rightarrow f(1) = 5$

$x^2 + 2x \xrightarrow{x \geq 1} f(1) = 5$ و $2x + a + 2 \xrightarrow{x < 1} f(1) = 5$

$|x+1| \geq 2 \Rightarrow \begin{cases} x+1 \geq 2 \Rightarrow x \geq 1 \\ x+1 \leq -2 \Rightarrow x \leq -3 \end{cases} \rightarrow f(x) = \begin{cases} x^2 - b & x \geq 1 \text{ و } x \leq -3 \\ a + 2x - x^2 & -3 < x < 1 \end{cases}$

$f(-2) = 2(-2)^2 - b = a + 2(-2) \Rightarrow 18 - b = a - 4 \Rightarrow a + b = 22$

$a + b = 22$

الف) $y = \frac{1}{x^2 - 4x} \rightarrow x^2 - 4x \neq 0 \Rightarrow x^2(x-4) \neq 0 \Rightarrow x \neq 0 \text{ و } x \neq 4 \Rightarrow D_f = \mathbb{R} - \{0, 4\}$

ب) $y = \frac{1}{2x^3 + 2x^2 + 2x + 2} \rightarrow 2x^3 + 2x^2 + 2x + 2 \neq 0 \Rightarrow 2x(x^2 + x + 1) + 2 \neq 0 \Rightarrow$

$(2x+2)(x^2+1) \neq 0 \Rightarrow \begin{cases} 2x \neq -2 \Rightarrow x \neq -1 \\ x^2 + 1 \neq 0 \end{cases} \rightarrow D_f = \mathbb{R} - \{-1\}$

$$\text{الف) } y = \sqrt{x^2-1} + \sqrt{x-2} \quad \left. \begin{array}{l} \text{I) } x^2-1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow -1 \leq x, x \geq 1 \\ \text{II) } x-2 \geq 0 \Rightarrow x \geq 2 \end{array} \right\} \xrightarrow{\text{I} \cap \text{II}} x \geq 2 \Rightarrow D_f = [2, +\infty)$$

$$\text{ب) } y = \frac{\sqrt{4-3x}}{\sqrt{9x-2}} \quad \left. \begin{array}{l} \text{I) } 4-3x \geq 0 \Rightarrow 3x \leq 4 \Rightarrow x \leq \frac{4}{3} \\ \text{II) } 9x-2 \geq 0 \Rightarrow 9x \geq 2 \Rightarrow x \geq \frac{2}{9} \end{array} \right\} \xrightarrow{\frac{2}{9} \leq x \leq \frac{4}{3}} D_f = \left[\frac{2}{9}, \frac{4}{3}\right]$$

$$y = \sqrt{4x-a} + \sqrt{b-3x}, D_f = \{x\}$$

$$\text{I) } 4x-a \geq 0 \Rightarrow 4x \geq a \Rightarrow x \geq \frac{a}{4}$$

$$\text{II) } b-3x \geq 0 \Rightarrow b \geq 3x \Rightarrow x \leq \frac{b}{3} \longrightarrow D_f = \left[\frac{a}{4}, \frac{b}{3}\right] = \{x\}$$

$$\frac{a}{4} = \frac{b}{3} = 2 \Rightarrow a=8 \text{ و } \frac{b}{3} = 2 \Rightarrow b=6 \Rightarrow \boxed{\frac{b}{a} = \frac{6}{8} = \frac{3}{4}}$$

$$\text{الف) } y = \sqrt{\frac{x^2-x-6}{x^2-13x+42}} \xrightarrow{\text{تجزیه صورت و مخرج}} \frac{(x-3)(x+2)}{(x+2)(x-7)(x-2)(x+3)} \Rightarrow \frac{-3}{+3-3-3+3} = \frac{-3}{-3} = 1 \Rightarrow$$

$$x^2-x-6 \geq 0 \Rightarrow (x-3)(x+2) \geq 0$$

$$D_f = (-\infty, -2) \cup (2, 3) \cup (3, +\infty)$$

$$x^2-13x+42 \geq 0 \Rightarrow x^2-13x+42 = 0 \Rightarrow (x-4)(x-9) = 0 \Rightarrow$$

$$x^2-4 \geq 0 \Rightarrow x = \pm 2$$

$$x^2-9 \geq 0 \Rightarrow x = \pm 3$$

$$\text{ب) } y = \sqrt{\frac{x^2-2x^2-5x+7}{x^2+2x^2-5x-6}} \Rightarrow \frac{x^2-2x^2-5x+7}{x^2+2x^2-5x-6} = \frac{-x^2-5x+7}{3x^2-5x-6} \Rightarrow \frac{-1}{3} = \frac{-1}{3} = 1 \Rightarrow$$

$$D_f = (-\infty, -2) \cup (-2, -1) \cup (1, 2) \cup (2, +\infty)$$

چون صفرهای صورت و مخرج برابرند پس صورت را می‌توانیم حذف کنیم و به صورت زیر می‌آید: $(x-1)(x^2-x-6) = (x-1)(x-3)(x+2)$ و مخرج $(x+1)(x^2+x-6) = (x+1)(x+3)(x-2)$

$$\text{الف) } y = \sqrt{2[x]-4} \Rightarrow 2[x] \geq 4 \Rightarrow [x] \geq 2 \Rightarrow x \geq 2 \Rightarrow D_f = [2, +\infty)$$

$$\text{ب) } y = \sqrt{2[x]-5} \Rightarrow 2[x] \geq 5 \Rightarrow [x] \geq \frac{5}{2} \Rightarrow x \geq \frac{5}{2} \Rightarrow D_f = \left[\frac{5}{2}, +\infty\right)$$

$$\text{الف) } y = \sqrt{\frac{x+1}{1x-21}} + \sqrt{\frac{4-x}{x^2+2x+1}} \Rightarrow \text{I) } \frac{-1}{49} = \frac{1}{49} \Rightarrow \frac{-1}{49} = \frac{1}{49} \Rightarrow \text{II) } \frac{4}{9} = \frac{4}{9} \Rightarrow \frac{4}{9} = \frac{4}{9} \Rightarrow \text{I} \cap \text{II} \Rightarrow$$

$$\Rightarrow D_f = (-\infty, -1] \cup (3, 4]$$

$$\text{ب) } y = \sqrt{[x]-4} + \sqrt{2-[x]} \Rightarrow \text{I) } [x] \geq 4 \Rightarrow x \geq 4 \quad \text{II) } [x] \leq 2 \Rightarrow x \leq 2 \xrightarrow{\text{I} \cap \text{II}} \Rightarrow D_f = \emptyset$$