

۲. افزایش ضریب عالی در صورت نوشتن

الف) $y = \frac{x^2+3}{2x^3+3x^2-18x+3}$ مخرج کسر باید مخالف صفر باشد مجموع ضرایب مخرج برابر صفر است پس بر ۱ است تقسیم بر $(x-1)$

$$2x^3 + 3x^2 - 18x + 3 = 0 \rightarrow (x-1)(2x^2 + 5x - 3) = 0$$

$$\textcircled{1} x-1=0 \rightarrow x=1$$

$$\textcircled{2} 2x^2 + 5x - 3 = 0 \rightarrow (2x-1)(x+3) = 0 \rightarrow \begin{cases} x = \frac{1}{2} \\ x = -3 \end{cases}$$

$D_f = \mathbb{R} - \left\{ -\frac{3}{2}, 1, 3 \right\}$

ب) $y = \frac{x+3}{2x^3+9x^2+10x+3}$ مخرج کسر باید مخالف صفر باشد جمع ضرایب توان های زوج برابر صفر است پس بر ۱ است تقسیم بر $(x+1)$

$$2x^3 + 9x^2 + 10x + 3 = 0 \rightarrow (x+1)(2x^2 + 7x + 3) = 0$$

$$\textcircled{1} x+1=0 \rightarrow x=-1$$

$$\textcircled{2} 2x^2 + 7x + 3 = 0 \rightarrow (2x+3)(x+1) = 0 \rightarrow \begin{cases} x = -\frac{3}{2} \\ x = -1 \end{cases}$$

$D_f = \mathbb{R} - \left\{ -\frac{3}{2}, -1, \frac{1}{2} \right\}$

الف) $y = \frac{x+3}{x^3-2x^2+2x-1}$ مخرج کسر باید مخالف صفر باشد مجموع ضرایب مخرج برابر صفر است پس بر ۱ است تقسیم بر $(x-1)$

$$x^3 - 2x^2 + 2x - 1 = 0 \rightarrow (x-1)(x^2 - x + 1) = 0$$

$$\textcircled{1} x-1=0 \rightarrow x=1$$

$$\textcircled{2} x^2 - x + 1 = 0 \rightarrow \Delta = b^2 - 4ac = 1 - 4 = -3 < 0 \rightarrow \Delta < 0 \rightarrow \text{ریشه حقیقی ندارد}$$

$D_f = \mathbb{R} - \{1\}$

ب) $y = \frac{x+3}{\sqrt{x^3-2x^2+2x-1}}$ مخرج کسر باید مخالف صفر باشد تقسیم بر $(x-1)$ تقسیم بر $(x-1)$

$$x^3 - 2x^2 + 2x - 1 \geq 0 \rightarrow (x-1)(x^2 - x + 1) \geq 0$$

$$\textcircled{1} x-1=0 \rightarrow x=1$$

$$\textcircled{2} x^2 - x + 1 = 0 \rightarrow \Delta = 1 - 4 = -3 < 0$$

$D_f = (-\infty, -3] \cup (1, +\infty)$

الف) $y = \frac{1}{x^2 - 5|x-1| - 2x + 5}$ ابتدا ضابطه بندی می کنیم

$$\begin{cases} x \geq 1 \rightarrow x^2 - 5x + 5 - 2x + 5 = x^2 - 7x + 10 \\ x < 1 \rightarrow x^2 + 5x - 5 - 2x + 5 = x^2 + 3x \end{cases}$$

مخرج نباید صفر شود

$$\begin{cases} x \geq 1 \rightarrow x^2 - 7x + 10 = 0 \rightarrow (x-2)(x-5) = 0 \rightarrow \begin{cases} x=2 \\ x=5 \end{cases} \\ x < 1 \rightarrow x^2 + 3x = 0 \rightarrow x(x+3) = 0 \rightarrow \begin{cases} x=0 \\ x=-3 \end{cases} \end{cases}$$

$D_f = \mathbb{R} - \{ -3, 0, 2, 5 \}$

الف) $y = \sqrt{-x} - 2 \rightarrow [-x] \geq 2 \rightarrow -x \geq 2 \rightarrow x \leq -2 \rightarrow D_f = (-\infty, -2]$

ب) $y = \frac{5x^2+4}{[x]^2-2[x]-3}$ مخرج کسر باید مخالف صفر باشد تقسیم بر $([x]-3)$

$$[x]^2 - 2[x] - 3 = 0 \rightarrow ([x]-3)([x]+1) = 0$$

$$\begin{cases} [x]=3 \rightarrow x = [3, 4) \\ [x]=-1 \rightarrow x = [-1, 0) \end{cases} \rightarrow D_f = (-\infty, -1) \cup [0, 3) \cup [4, +\infty)$$

الف) $y = \sqrt{[x-\frac{1}{2}]} + [x+\frac{1}{2}] \rightarrow [x-\frac{1}{2}] + [x+\frac{1}{2}] \geq 0 \rightarrow [x-\frac{1}{2}] + [x-\frac{1}{2}+1] \geq 0$

$$2[x-\frac{1}{2}] \geq -1 \rightarrow [x-\frac{1}{2}] \geq -\frac{1}{2} \rightarrow x-\frac{1}{2} \geq 0 \rightarrow x \geq \frac{1}{2} \rightarrow D_f = [\frac{1}{2}, +\infty)$$

ب) $y = \sqrt{[x-\frac{1}{2}]} + [\frac{1}{2}-x] \rightarrow [x-\frac{1}{2}] + [-x-\frac{1}{2}] \geq 0$ if $\begin{cases} a \in \mathbb{Z} & [a] + [-a] = 0 \\ a \notin \mathbb{Z} & [a] + [-a] = -1 \end{cases}$

$$x - \frac{1}{2} \in \mathbb{Z} \rightarrow D_f = \{ x \mid x = k + \frac{1}{2}, k \in \mathbb{Z} \}$$

$a) y = \frac{\cot x + 1}{\tan x + 1} \rightarrow \begin{cases} \tan x \neq -1 \rightarrow x \neq \{k\pi - \frac{\pi}{4}\} \\ \cos x \neq 0 \rightarrow x \neq \{k\pi + \frac{\pi}{2}\} \\ \sin x \neq 0 \rightarrow x \neq \{k\pi\} \end{cases} \rightarrow D_f = \mathbb{R} - \left\{ \frac{k\pi}{4}, k\pi - \frac{\pi}{4} \right\}$ 	6
$b) y = \sqrt{1 - F \sin^2 x} \rightarrow 1 - F \sin^2 x \geq 0 \rightarrow F \sin^2 x \leq 1 \rightarrow \sin^2 x \leq \frac{1}{F} \rightarrow \frac{1}{F} \leq \sin x \leq \frac{1}{F}$ $\rightarrow \frac{1}{F} \leq \sin x \leq \frac{1}{F} \rightarrow D_f = \left[k\pi - \frac{\pi}{2}, k\pi + \frac{\pi}{2} \right]$ 	7
$a) y = \sqrt{F \sin x - 1} \rightarrow F \sin x - 1 \geq 0 \rightarrow \sin x \geq \frac{1}{F} \rightarrow D_f = \left[k\pi + \frac{\pi}{2}, k\pi + \frac{\pi}{2} \right]$ $\frac{\pi}{2} = \frac{\pi}{2} \rightarrow x = \frac{\pi}{2}$ 	8
$b) \sqrt{1 - F \cos x} \rightarrow 1 - F \cos x \geq 0 \rightarrow F \cos x \leq 1 \rightarrow \cos x \leq \frac{1}{F}$ $\rightarrow D_f = \left[k\pi + \frac{\pi}{2}, k\pi + \frac{\pi}{2} \right]$ 	9
$a) y = \arcsin(x^2 + Fx + 1) \rightarrow -1 \leq x^2 + Fx + 1 \leq 1 \rightarrow \begin{cases} x^2 + Fx + 1 \geq -1 \rightarrow x^2 + Fx + 2 \geq 0 \rightarrow (x + F/2)^2 \geq 0 \text{ (I)} \\ x^2 + Fx + 1 \leq 1 \rightarrow x^2 + Fx \leq 0 \rightarrow x(x + F) \leq 0 \text{ (II)} \end{cases}$ $\rightarrow x \in \left[-\frac{F}{2}, \frac{F}{2} \right] \text{ (I)} \quad x \in \left[-F, 0 \right] \text{ (II)} \rightarrow D_f = (I) \cap (II) \rightarrow D_f = \left[-F, \frac{F}{2} \right]$	10
$b) \cos y = \sqrt{\omega - Fx} \rightarrow \begin{cases} \omega - Fx \geq 0 \rightarrow Fx \leq \omega \rightarrow x \leq \frac{\omega}{F} \text{ (I)} \\ -1 \leq \sqrt{\omega - Fx} \leq 1 \rightarrow \omega - Fx \leq 1 \rightarrow Fx \geq \omega - 1 \rightarrow x \geq \frac{\omega - 1}{F} \text{ (II)} \end{cases}$ $\rightarrow D_f = (I) \cap (II) \rightarrow D_f = \left[\frac{\omega - 1}{F}, \frac{\omega}{F} \right]$	11
$a) y = \log_{\frac{1}{F}}(1 - \log_{\frac{1}{F}} x) \rightarrow \begin{cases} x > 0 \text{ (I)} \\ 1 - \log_{\frac{1}{F}} x > 0 \rightarrow \log_{\frac{1}{F}} x < 1 \rightarrow x < F \text{ (II)} \end{cases} \rightarrow D_f = (I) \cap (II) \rightarrow D_f = (0, F)$	12
$b) y = \log_{\frac{1}{F}}(1 - \log_{\frac{1}{F}} \frac{x}{F}) \rightarrow \begin{cases} x > 0 \text{ (I)} \\ 1 - \log_{\frac{1}{F}} \frac{x}{F} > 0 \rightarrow \log_{\frac{1}{F}} \frac{x}{F} < 1 \rightarrow -\log_{\frac{1}{F}} x < 1 \rightarrow \log_{\frac{1}{F}} x > -1 \rightarrow x > F^{-1} \rightarrow x > \frac{1}{F} \text{ (II)} \end{cases}$ $\rightarrow D_f = (I) \cap (II) \rightarrow D_f = \left(\frac{1}{F}, +\infty \right)$	13
$a) y = \sqrt{F^2 x - x^2} - 1 \rightarrow F^2 x - x^2 - 1 \geq 0 \rightarrow F^2 x - x^2 \geq 1 \rightarrow F^2 x - x^2 - 1 \geq 0 \rightarrow (x - F)(x - 1) \leq 0 \rightarrow x \in [1, F]$	14
$b) y = \left(\frac{F^2 x + \omega}{F^2 x + F} \right)! \rightarrow \frac{F^2 x + \omega}{F^2 x + F} \in \mathbb{W} \rightarrow F^2 x + \omega \in F\mathbb{W} \rightarrow F^2 x - F\mathbb{W} \in F\mathbb{W} - \omega \rightarrow x \in \frac{F\mathbb{W} - \omega}{F^2 - F\mathbb{W}}$ $\rightarrow D_f = \left\{ x \mid x = \frac{FK - \omega}{F^2 - FK}, K \in \mathbb{W} \right\}$	15