

۲. اثبات عدلی و کامل نوشت

الف) $y = \frac{x+3}{x^3+3x^2-18x+3}$ تجزیه بر $(x-1)$ $\rightarrow x^3+3x^2-18x+3 = (x-1)(2x-1)(x+3) \rightarrow$ ریشه ها $= -3, \frac{1}{2}, 1$
 $\frac{-3}{\frac{1}{2}+3} + \frac{1}{\frac{1}{2}-1} + \frac{1}{1-1} \rightarrow$ جواب: $D = \mathbb{R} - \left\{ -3, \frac{1}{2}, 1 \right\}$

ب) $y = \frac{x+3}{x^3+9x^2+10x+3}$ تجزیه بر $(x+1)$ $\rightarrow x^3+9x^2+10x+3 = (x+3)(x+1)(x+1) \rightarrow$ ریشه ها $= -3, -1, -1$
 $\rightarrow$ جواب: $D = \mathbb{R} - \left\{ -3, -1 \right\}$

الف) $y = \frac{x+3}{x^3-2x^2+2x-1}$ تجزیه بر $(x-1)$ $\rightarrow (x-1)(x^2-x+1)$ $\rightarrow D = \mathbb{R} - \{1\} \subseteq (-\infty, 1) \cup (1, +\infty)$
 جواب: $\rightarrow$

ب) $y = \sqrt{\frac{x+3}{x^3-2x^2+2x-1}}$ $\rightarrow \frac{x+3}{(x-1)(x^2-x+1)} \geq 0 + \frac{-3}{\frac{1}{2}-1} + \frac{1}{1-1} \rightarrow D = (-\infty, -3] \cup (1, +\infty)$
 جواب: $\rightarrow$

$y = \frac{2}{x^2-a|x-1|-2x+a}$ $\rightarrow x^2-a|x-1|-2x+a \neq 0 \rightarrow x^2-2x+a-a|x-1| \neq 0$
 $\rightarrow (x^2-2x+1)+\varepsilon-a|x-1| \neq 0 \rightarrow \underbrace{(x-1)^2}_{a}-a|x-1|+\varepsilon \neq 0 \rightarrow a^2-a+\varepsilon \neq 0$
 $\rightarrow (a-\varepsilon)(a-1) \neq 0 \rightarrow a \neq 1, a \neq 1 \rightarrow a = |x-1| \rightarrow |x-1| \neq 1 \rightarrow x-1 \neq 1 \rightarrow x \neq 2$
 $|x-1| \neq \varepsilon \rightarrow x-1 \neq \varepsilon \rightarrow x \neq 1+\varepsilon \rightarrow x \neq -3$ $\rightarrow D = \mathbb{R} - \{2, -3, 1\}$

الف) $y = \sqrt{[x]-2}$ $\rightarrow [x] \geq 2 \rightarrow -x \geq 2 \rightarrow x \leq -2 \rightarrow D = (-\infty, -2]$
 جواب: $\rightarrow$

ب) $y = \frac{5x^2+6}{[x]^2-2[x]-3}$ $\rightarrow ([x]-3)([x]+1) \rightarrow [x] \neq 3, -1$
 $[x]^2-2[x]-3 \neq 0 \rightarrow D = (-\infty, -1) \cup (3, +\infty)$
 جواب: $\rightarrow$

الف) $y = \sqrt{\left[x-\frac{1}{3}\right] + \left[x+\frac{2}{3}\right]} = \left[x+\frac{2}{3}-1\right] + \left[x+\frac{2}{3}\right] \geq 0 \rightarrow 2\left[x+\frac{2}{3}\right] \geq 1 \rightarrow \left[x+\frac{2}{3}\right] \geq \frac{1}{2}$
 $\rightarrow x+\frac{2}{3} \geq 1 \rightarrow D = \left[\frac{1}{3}, +\infty\right)$
 جواب: $\rightarrow$

ب) $y = \sqrt{\left[x-\frac{1}{3}\right] + \left[\frac{1}{3}-x\right]}$ $t = x-\frac{1}{3}, -t = \left[\frac{1}{3}-x\right] \rightarrow [t]+[-t] = 0 \rightarrow$ $x-\frac{1}{3} \in \mathbb{Z} \rightarrow D = \left\{ \frac{1}{3}+n \mid n \in \mathbb{Z} \right\}$
 جواب: $\rightarrow$

ان) $y = \frac{\cos x + 1}{\tan x + 1} \rightarrow \begin{cases} \cos x \neq 0 \\ \sin x \neq 0 \\ \tan x \neq -1 \end{cases} \rightarrow \text{Unit Circle} \rightarrow \mathcal{D} = \mathbb{R} - \left\{ \frac{k\pi}{2}, k\pi - \frac{\pi}{4} \right\}$

ب) $y = \sqrt{1 - \varepsilon \sin^2 x} \rightarrow 1 - \varepsilon \sin^2 x \geq 0 \rightarrow \sin^2 x \leq \frac{1}{\varepsilon} \rightarrow \sin^2 x \leq \frac{1}{4} \rightarrow -\frac{1}{2} \leq \sin x \leq \frac{1}{2} \rightarrow \mathcal{D} = \left[k\pi - \frac{\pi}{6}, k\pi + \frac{\pi}{6} \right]$

ج) $y = \sqrt{2 \sin x - 1} \rightarrow 2 \sin x - 1 \geq 0 \rightarrow \sin x \geq \frac{1}{2} \rightarrow \text{Unit Circle} \rightarrow \mathcal{D} = \left[2k\pi + \frac{\pi}{6}, 2k\pi + \frac{5\pi}{6} \right]$

د) $y = \sqrt{1 - 2 \cos x} \rightarrow 1 - 2 \cos x \geq 0 \rightarrow \cos x \leq \frac{1}{2} \rightarrow \text{Unit Circle} \Rightarrow \mathcal{D} = \left[2k\pi + \frac{\pi}{3}, 2k\pi + \frac{5\pi}{3} \right]$

هـ) $y = \arcsin(x^2 + 2x + 1) \rightarrow -1 \leq x^2 + 2x + 1 \leq 1 \rightarrow x^2 + 2x \leq 0 \rightarrow x(x+2) \leq 0 \rightarrow x \in [-2, 0] \rightarrow \mathcal{D} = [-2, 0]$

و) $y = \arcsin(\sqrt{a - 2x}) \rightarrow a - 2x \geq 0 \rightarrow 2x \leq a \rightarrow x \leq \frac{a}{2} \rightarrow -1 \leq \sqrt{a - 2x} \leq 1 \rightarrow a - 2x \leq 1 \rightarrow -2x \leq 1 - a \rightarrow -2(x - 2) \leq \frac{1}{2} \rightarrow x \geq 2 \rightarrow \mathcal{D} = \left[2, \frac{a}{2} \right]$

ز) $y = \log_2(1 - \log_3 x) \rightarrow x > 0 \rightarrow 1 - \log_3 x > 0 \rightarrow \log_3 x < 1 \rightarrow x < 3 \rightarrow \mathcal{D} = (0, 3)$

ح) $y = \log_2(1 - \log_{\frac{1}{3}} x) \rightarrow x > 0 \rightarrow 1 - \log_{\frac{1}{3}} x > 0 \rightarrow \log_{\frac{1}{3}} x < 1 \rightarrow \frac{1}{x} < 3 \rightarrow x > \frac{1}{3} \rightarrow \mathcal{D} = \left(\frac{1}{3}, +\infty \right)$

ط) $y = \sqrt{2^{4x - x^2} - 1} \rightarrow 2^{4x - x^2} \geq 1 \rightarrow 2^{4x - x^2} \geq 2^0 \rightarrow 4x - x^2 \geq 0 \rightarrow x(4 - x) \geq 0 \rightarrow x \in [0, 4] \rightarrow \mathcal{D} = [0, 4]$

ي) $\left(\frac{3x + a}{2x + \varepsilon} \right) > 0 \rightarrow \frac{3x + a}{2x + \varepsilon} \in \mathbb{W} \rightarrow x = \frac{fk - a}{k - 2k} \rightarrow \mathcal{D} = \left\{ x \in \mathbb{R} \mid \frac{fk - a}{k - 2k}, k \in \mathbb{W} \right\}$