

۲. آخرین معادله نوشتاری

معادله را در دو طرف با x ضرب می‌کنیم

$$-\infty < x < \infty \Rightarrow a-x \leq 0 \Rightarrow a \leq x$$

$$\Rightarrow 2a + b \xrightarrow{x \leq c} 4 + b \leq 0 \Rightarrow b \leq -4$$

$$\Rightarrow x^2 + 2x + 1 - m^2 \rightarrow \text{معادله ناقص}$$

$$\Rightarrow 0 \leq 0 \Rightarrow x - 1 + 2m^2 \leq 0 \Rightarrow x(m^2 - 1) \leq 0 \Rightarrow m^2 - 1 < 0 \Rightarrow -1 \leq m \leq 1$$

$$\Rightarrow 1 - (-1) = 2$$

$$x - m^2 \geq 0 \Rightarrow x \geq m^2 \Rightarrow -1 \leq x \leq 1$$

$$[x] \neq 0 \Rightarrow [-x] \neq 0 \rightarrow \dots \rightarrow x \in \mathbb{Z} \Rightarrow x \in \{-2, -1, 0, 1, 2\}$$

$$\text{حل) } y = \sqrt{x^2 + c} \Rightarrow x^2(x+c) \geq 0 \Rightarrow \frac{-c}{-1 \pm 1} \Rightarrow P = \{c, \infty\}$$

$$\therefore y = \sqrt{\frac{(x^2-1)(x^2-cx^2)}{x^2-1}} = \sqrt{\frac{(x-1)(x+1)(x-c)x^2}{(x-1)(x^2+x+1)}} \Rightarrow \frac{-1}{0} \quad \frac{0}{-1} \quad \frac{1}{0} \quad \frac{c}{1}$$

$$\Rightarrow P = \{-\infty, -1\} \cup \{0\} \cup [c, \infty)$$

$$\text{حل) } y = \sqrt{|x-1|-c} \Rightarrow |x-1|-c \geq 0 \Rightarrow |x-1| \geq c \Rightarrow x-1 \geq c \Rightarrow x \geq c+1$$

$$\Rightarrow x-1 \leq -c \Rightarrow x \leq -1$$

$$\Rightarrow P = \{-\infty, -1\} \cup [c+1, \infty)$$

$$\therefore y = \sqrt{4 - |2-x|} \Rightarrow |2-x| \leq 4 \Rightarrow -4 \leq 2-x \leq 4 \Rightarrow 4 \geq x \geq -2 \Rightarrow P = [-2, 4]$$

$$\text{حل) } y = \sqrt{1 - x|x|} \Rightarrow x|x| \leq 1 \Rightarrow \begin{cases} x \leq 0 \Rightarrow -x^2 \leq 1 \Rightarrow \text{همیشه} \\ x > 0 \Rightarrow x^2 \leq 1 \Rightarrow -1 \leq x \leq 1 \Rightarrow 0 < x \leq 1 \end{cases}$$

$$\Rightarrow P = (-\infty, 1]$$

$$\therefore y = \frac{1}{\sqrt{c - |x-1|} - 1} \Rightarrow |x-1| \leq c \Rightarrow -c \leq x-1 \leq c \Rightarrow 1-c \leq x \leq 1+c$$

$$\Rightarrow \sqrt{c - |x-1|} \neq 1 \Rightarrow c - |x-1| \neq 1 \Rightarrow x \neq 1 \pm \sqrt{c-1}$$

$$\Rightarrow P = [1-c, 1+c] - \{1 \pm \sqrt{c-1}\}$$

$$a) y = \frac{\sin x - 1}{\cos x} \Rightarrow \cos x \neq 0 \Rightarrow P_f = \mathbb{R} - \left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\} \quad -v$$

$$b) y = \frac{\cos x}{1 + \cos x} \Rightarrow \cos x \neq -1 \Rightarrow P_f = \mathbb{R} - \left\{ \frac{\pi}{2}, \frac{3\pi}{2}, \pi \right\}$$

$$a) y = \frac{x^2}{|x+1| - |x-1|} \Rightarrow |x+1| \neq |x-1|$$

$$\Rightarrow x^2 - 4x + 9 \neq 0 \Rightarrow (x^2 - 4x + 4) + 5 \neq 0$$

$$\Rightarrow (x-2)^2 + 5 \neq 0$$

$$\Rightarrow x \neq -\frac{1}{2} \Rightarrow P_f = \mathbb{R} - \left\{ -\frac{1}{2} \right\}$$

$$b) y = \sqrt{|x+1| - |x-1|} \Rightarrow |x+1| \geq |x-1| \Rightarrow x^2 - 4x + 9 \geq 0$$

$$\Rightarrow (x-2)^2 - 4 + 9 \geq 0 \Rightarrow (x-2)^2 + 5 \geq 0 \Rightarrow P_f = (-\infty, -\frac{1}{2}] \cup [2, \infty)$$

$$|x-1| > 0 \Rightarrow x > 1 \Rightarrow x > 0$$

$$\log_2(x-1) > 0 \Rightarrow x-1 > 1 \Rightarrow x > 2$$

$$\log_2(x-1) < 1 \Rightarrow x-1 < 2 \Rightarrow x < 3$$

$$\log_2(x-1) \leq 1 \Rightarrow x-1 \leq 2 \Rightarrow x \leq 3$$

$$P_f = (1, 3]$$

$$a) y = \log_2(\cos x + 1) \Rightarrow \cos x + 1 > 0 \Rightarrow \cos x > -1 \Rightarrow$$

$$P_f = \left(\frac{\pi}{2}, \frac{3\pi}{2} \right)$$

$$b) y = \sqrt{\log \frac{x-1}{x+1}} \Rightarrow \frac{x-1}{x+1} > 0 \Rightarrow \frac{-1}{\frac{1}{0} - \frac{1}{0}}$$

$$\Rightarrow \log \frac{x-1}{x+1} > 0 \Rightarrow \frac{x-1}{x+1} > 1 \Rightarrow \frac{x-1-x-1}{x+1} > 0 \Rightarrow \frac{-2}{x+1} > 0$$

$$\Rightarrow P_f = (-\infty, -1)$$