

$$f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \Rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \Rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0$$

$$\Rightarrow \frac{1-2x}{x(x-1)} \geq 0 \Rightarrow \frac{0}{-} \quad \frac{1}{+} \quad \frac{1}{-} \Rightarrow D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$$

$$f(x) = \frac{1}{x+1} - \frac{x}{x-1} \Rightarrow x \neq -1, x \neq 0$$

$$\frac{x}{x-1} + \frac{1}{x+1} \neq 0 \Rightarrow x \neq 1, x \neq -2$$

$$\frac{3x+4+x-1}{(x-1)(x+2)} \neq 0 \Rightarrow \frac{4x+3}{(x-1)(x+2)} \neq 0 \Rightarrow x \neq -\frac{3}{4} \Rightarrow D_f = \mathbb{R} - \{-2, -\frac{3}{4}, -1, 0, 1\}$$

$$f(x) = \sqrt{\left[\left(\frac{1}{x}\right)^x - 9\right] (3x - x^x)} \Rightarrow \left(\left(\frac{1}{x}\right)^x - 9\right) (3x - x^x) \geq 0 \Rightarrow \begin{cases} 3x = x^x \\ = (x^x)^{\frac{1}{x}} \\ = x^{\frac{x}{x}} \end{cases}$$

$$\frac{-3}{+} \quad \frac{0}{-} \quad \frac{0}{+} \Rightarrow D_f = (-\infty, -3] \cup [\frac{0}{x}, +\infty)$$

$$f(x) = \sqrt{x-1} + \sqrt{y+1} = 3 \Rightarrow \sqrt{x-1} = \sqrt{y+1} \Rightarrow 3 - \sqrt{x-1} \geq 0$$

$$\Rightarrow \sqrt{x-1} \leq 3 \Rightarrow x-1 \leq 9 \Rightarrow x \leq 10$$

$$\Rightarrow D_f = [1, 10]$$

$$f(x) = \log_{\frac{1}{2}} (x^2 - x - 2) \Rightarrow x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0 \Rightarrow \frac{-1}{+} \quad \frac{2}{-} \quad \frac{2}{+}$$

$$\frac{1}{\sqrt{x^2-1}} + 1 \Rightarrow x^2 - 1 \geq 0 \Rightarrow \frac{-1}{+} \quad \frac{1}{-} \quad \frac{1}{+} \Rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$$

$$f(x) = \sqrt{3+ax - x^2} \quad D_f = [-2, b] \rightarrow \frac{-2}{-} \quad \frac{b}{+} \quad \frac{b}{-}$$

$$\Rightarrow \sqrt{3-2a-x} = 0 \Rightarrow \sqrt{-(2a+1)} = 0 \Rightarrow a = -\frac{1}{2} \Rightarrow f(x) = \sqrt{-x^2 - \frac{1}{2}x + 3}$$

$$\Rightarrow -x^2 - \frac{1}{2}x + 3 \geq 0 \Rightarrow -2x^2 - x + 6 \geq 0 \Rightarrow x^2 - x - 12 \geq 0 \Rightarrow (x-4)(x+3) \geq 0$$

$$\Rightarrow x \begin{cases} \frac{4}{-} \quad \frac{-3}{-} \\ \frac{-3}{+} \quad \frac{4}{+} \end{cases} = -2 \quad \frac{4}{-} \quad \frac{-3}{-} = \frac{4}{2} = 2 = b$$

$$a+b = -\frac{1}{2} + \frac{3}{2} = \frac{2}{2} = 1$$

$$f(u) = \begin{cases} 2u - 2 & u \geq 1 \\ 2u + 2 & u < 1 \end{cases}$$

$$g(u) = \sqrt{f(u) - u} \Rightarrow f(u) - u \geq 0$$

$$\begin{array}{lcl} & u \geq 1 & u < 1 \\ \swarrow & & \searrow \\ 2u - 2 \geq 0 & & 2u + 2 \geq 0 \\ 2(u - 1) \geq 0 & & u \geq -1 \\ u \geq 1 & & u \geq -1 \\ \Downarrow & & \Downarrow \\ u \geq 1 & & [-1, 1) \\ [1, +\infty) & \cup & \end{array}$$

$$\Rightarrow D_g = [-1, +\infty)$$

$$\begin{aligned} f(u) &= 2u^2 - \varepsilon u & f\left(\sin \frac{\pi}{4} + \cos 2\pi\right) &= f\left(\frac{\sqrt{2}}{2} + 1\right) = f\left(\frac{1+\sqrt{2}}{2}\right) \\ \Rightarrow f(u) &= u(2u - \varepsilon) & &= \frac{1+\sqrt{2}}{2} (2 + \sqrt{2} - \varepsilon) = \frac{1+\sqrt{2}}{2} \times \sqrt{2} - 2 = \frac{2-\varepsilon}{2} = -\frac{1}{2} \end{aligned}$$

$$f(u) = \frac{u+1}{u-1}$$

$$\begin{aligned} f(f(a)) &= a \Rightarrow f\left(\frac{a+1}{a-1}\right) = a \Rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = a \Rightarrow \frac{\frac{2a}{a-1}}{\frac{2}{a-1}} = a \Rightarrow 1, \sqrt{a} \\ \Rightarrow a &= a & \Rightarrow f(f(a)) & \text{ به ازای هر مقدار معادله صحیح است.} \\ & & & \text{به ازای } a=1 \text{ عبارت تعریف نمی شود!} \\ & & & D_f = \mathbb{R} - \{1\} \end{aligned}$$

$$\begin{aligned} f(u) &= u^2 - \varepsilon u \\ g(u) &= \sqrt{u^2 - 1} \end{aligned}$$

$$f(g(a)) = -\varepsilon \Rightarrow (g(a))^2 - g(a) = -\varepsilon$$

$$\Rightarrow [g(a)]^2 - \varepsilon g(a) + \varepsilon = 0$$

$$\Rightarrow (g(a) - 2)^2 = 0 \Rightarrow g(a) = 2 \Rightarrow \sqrt{a^2 - 1} = 2$$

$$\Rightarrow a^2 - 1 = 4 \Rightarrow a = \pm \sqrt{5}$$

$$\begin{aligned} f(u+2) &= u^2 - 4u + 4 \\ f(u+2) &= (u-2)(u-2) \end{aligned}$$

$$\begin{aligned} f(4) \times f(-2) &= f(1+2) \times f(-1+2) \\ &= -2 \times (-1) \times (-1) \times (-2) = 144 \end{aligned}$$

$$f(\varepsilon \cos^r \eta + \delta \cos \eta) = \tan^r$$

$$\frac{1}{\cos^r \eta} - 1$$

$$\varepsilon \cos^r \eta + \delta \cos \eta - r, \delta = 0$$

$$\varepsilon t^r + \delta t - r, \delta = 0 \Rightarrow t^r + \delta t - r = 0 \Rightarrow (t+r)(t-r) = 0 \Rightarrow \cos \eta = t$$

$\frac{r}{\varepsilon} = \frac{1}{r}$
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 $-1 \leq \cos \eta \leq 1$

$$f(r, \delta) = \frac{1}{\left(\frac{1}{r}\right)^r} - 1 = \frac{1}{\frac{1}{\varepsilon}} - 1$$

$\varepsilon - 1 = r$

$$f(f(a)) = \frac{f(a)+1}{f(a)-1} = a \rightarrow \frac{\frac{a+1}{a-1}+1}{\frac{a+1}{a-1}-1} = \frac{a+r+a-r}{a-1} = \frac{2a}{r} = a$$

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پس به ازای همه نقاط در دایره f برقرار است ✓

$x \in D_f \rightarrow x \in \mathbb{R} - \{1\}$