

نام و نام خانوادگی: پاسخنامه تشریحی تکلیف شماره ۱۰۰ کلاس: یازدهم ریاضی - B

۱۰. آفلاین عمل نوشتن

$$f(x) = \sqrt{\frac{x-1}{x}} - \frac{x}{x-1} \rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \rightarrow \frac{x^2 - 2x + 1 - x^2}{x(x-1)} \geq 0$$

$$\rightarrow \frac{-2x + 1}{x(x-1)} \geq 0$$

تعیین علامت می‌کنیم

x	0	$\frac{1}{2}$	1
$+$	$-$	$+$	$-$

$$\rightarrow D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$$

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$$f(x) = \frac{1}{x+1} - \frac{x}{x-1} \rightarrow \frac{1}{x+1} + \frac{1}{x+1} \rightarrow \frac{2}{x+1} \neq 0 \rightarrow x \neq -1$$

$$\rightarrow D_f = \mathbb{R} - \{-1, \frac{1}{2}, 1, 0\}$$

$$\frac{x}{x-1} + \frac{1}{x+1} \neq 0 \rightarrow \frac{x(x+1) + (x-1)}{(x-1)(x+1)} \neq 0 \rightarrow \frac{x^2 + x + x - 1}{(x-1)(x+1)} \neq 0 \rightarrow \frac{x^2 + 2x - 1}{(x-1)(x+1)} \neq 0$$

$$\rightarrow x \neq \frac{-2 \pm \sqrt{4 + 4}}{2} = \frac{-2 \pm 2}{2} = 0, -1$$

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$$f(x) = \sqrt{\left(\frac{1}{x}\right)^2 - 9(x^2 - 4^x)} \rightarrow \left(\left(\frac{1}{x}\right)^2 - \left(\frac{1}{x}\right)^2\right)(x^2 - 4^x) \geq 0$$

تعیین علامت می‌کنیم

x	0	2	4
$+$	$-$	$+$	$-$

$$\rightarrow D_f = (-\infty, 0) \cup [2, 4) \cup (4, +\infty)$$

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$$\sqrt{x-1} + \sqrt{y+1} = 3 \rightarrow \begin{cases} ① x-1 \geq 0 \rightarrow x \geq 1 \\ ② \sqrt{y+1} = 3 - \sqrt{x-1} \rightarrow 3 - \sqrt{x-1} \geq 0 \rightarrow \sqrt{x-1} \leq 3 \end{cases}$$

تفاوت علامت می‌کنیم

x	1	10
$+$	$-$	$+$

$$\rightarrow D_f = [1, 10]$$

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$$f(x) = \frac{\log_2(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \rightarrow \begin{cases} ① x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0 \rightarrow x < -1 \text{ یا } x > 2 \\ ② x^2 - 1 \geq 0 \rightarrow x^2 \geq 1 \rightarrow x \geq 1 \text{ یا } x \leq -1 \end{cases}$$

تفاوت علامت می‌کنیم

x	-1	2
$+$	$-$	$+$

$$\rightarrow D_f = (-\infty, -1) \cup [2, +\infty)$$

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$$f(x) = \sqrt{3 + a^2 x^2 - x^2} \rightarrow 3 + a^2 x^2 - x^2 \geq 0 \rightarrow x^2 - a^2 x^2 - 3 = 0$$

نقاط انتهایی نامعنه

$$\rightarrow x^2 - a^2 x^2 - 3 = 0 \rightarrow x^2 - a^2 x^2 = 3 \rightarrow x^2(1 - a^2) = 3$$

یعنی x^2 و a^2 رابطه‌های عبارت زیر را دنبال هستند.

$$\rightarrow x^2 - a^2 x^2 = 3 \rightarrow x^2(1 - a^2) = 3 \rightarrow x^2 = \frac{3}{1 - a^2}$$

$$\rightarrow a + b = \frac{1}{x} + \frac{3}{x} = \frac{4}{x}$$

$$\rightarrow a + b = 1$$

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$$g(x) = \sqrt{f(x) - x} \rightarrow f(x) - x \geq 0 \rightarrow \begin{cases} ① x \geq 1 \rightarrow 3x - 2 - x \geq 0 \rightarrow 2x - 2 \geq 0 \rightarrow x \geq 1 \\ ② x < 1 \rightarrow 2x + 3 - x \geq 0 \rightarrow x + 3 \geq 0 \rightarrow x \geq -3 \end{cases}$$

تفاوت علامت می‌کنیم

x	-3	1
$+$	$-$	$+$

$$\rightarrow D_f = (-3, 1) \cup [1, +\infty)$$

$$f(x) = x^2 - 1 \rightarrow f(x) = x^2 - 1 \rightarrow f(x) = x^2 - 1 \rightarrow f(x) = x^2 - 1 \rightarrow f(x) = x^2 - 1$$

$$\rightarrow f(\sin x + \cos x) \rightarrow x = \sin x + \cos x = \sqrt{2} + 1$$

$$f\left(\frac{1}{x} + 1\right) = \left(\frac{1}{x} + 1 - 1\right)^2 - 1 = \left(\frac{1}{x}\right)^2 - 1 = \frac{1}{x^2} - 1 = \frac{1-x^2}{x^2}$$

$$\rightarrow f(\sin x + \cos x) = \frac{1}{x^2}$$

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$$f(x) = \frac{x+1}{x-1} \rightarrow f(a) = \frac{a+1}{a-1} \rightarrow f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{a+1+a-1}{a+1-a+1} = \frac{2a}{2} = a$$

$$\rightarrow f(f(a)) = \frac{a}{a-1} \rightarrow a-1 \neq 0 \rightarrow a \neq 1$$

$$\rightarrow a \in \mathbb{R} - \{1\} \subseteq a \neq 1 \subseteq (-\infty, 1) \cup (1, +\infty)$$

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$$f(g(a)) = -f \rightarrow f(\sqrt{a^2-1}) = -f \rightarrow \frac{\sqrt{a^2-1}}{t-\sqrt{a^2-1}} = -f \rightarrow t - \sqrt{a^2-1} = -f \rightarrow t - \sqrt{a^2-1} + f = 0$$

$$\rightarrow t = 1 \rightarrow \sqrt{a^2-1} = 1 \rightarrow a^2-1 = 1 \rightarrow a^2 = 2 \rightarrow a = \pm\sqrt{2}$$

$$\rightarrow a = \pm\sqrt{2}$$

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$$f(x+y) = x^2 - \omega x + y \rightarrow \begin{cases} f(x) \rightarrow x^2 - \omega x = f \rightarrow x = 1 \rightarrow f(x) = 1 - \omega + y = 1 \\ f(y) \rightarrow y^2 - \omega y = -f \rightarrow y = -y \rightarrow f(y) = y^2 - \omega y + y = y^2 \end{cases}$$

$$\rightarrow f(x) \times f(y) = 1 \times 1 = 1$$

$$\rightarrow f(x) \times f(y) = 1$$

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$$f(\cos x + \omega \cos x) = \tan x \rightarrow f \cos x + \omega \cos x = \tan x = \frac{1}{x}$$

$$t = \cos x \rightarrow f t + \omega t = \frac{1}{x} \rightarrow f t + \omega t - \frac{1}{x} = 0 \rightarrow \lambda t^2 + 10t - 1 = 0$$

$$\rightarrow t = \frac{1}{x} \rightarrow f\left(\frac{1}{x}\right) = \tan x \rightarrow \tan x = \frac{1 - \cos x}{\cos x}$$

$$\rightarrow \tan x = \frac{1 - \frac{1}{x}}{\frac{1}{x}} = \frac{x-1}{1} = x-1 \rightarrow f(x) = x-1$$

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